Modeling of IEEE 802.15.4 in a Cluster of Synchronized Sensor Nodes

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Abstract. In this paper we present a model of the IEEE 802.15.4 MAC protocol for a cluster of sensor nodes and discuss its performance. Our focus is on a special scenario where all nodes start their transmission synchronously, which is the most harmful case for CSMA/CA protocols. The influence of the cluster size and message length is investigated using a non-stationary analysis technique. We characterize the delays for transmitting/receiving data from the cluster members, the success probability, and discuss energy consumption issues.

Keywords: clustered sensor networks, IEEE 802.15.4, CSMA/CA, energy consumption.

1 Introduction

With the recent developments in Micro Electro Mechanical System (MEMS) technology, large-scale networks of integrated wireless sensor nodes have become available [1]. By deploying networks of sensors, information about behavior, conditions, and positions of entities in an environment are gathered and forwarded to a sink for further processing. The nodes are equipped with a sensing device, radio transmitter, and are usually battery operated. Since they are designed to operate autonomously, they must be able to set up a communication network in an ad-hoc manner and to adapt to changes in the network topology, when individual nodes may fail due to exhausted batteries. Conservation of energy is, thus, a key issue in the deployment of sensor networks. Most energy consumption is caused by the communication over the radio link [2].

Recently, several publications have shown the benefits of using clustering methods in order to prolong the lifetime of the network, e.g. [3, 4]. In clustered sensor networks, the sensor nodes do not transmit their collected data to the sink, but to designated cluster heads which aggregate the data packets and send them directly or via multi-hop communication to the sink. Thus, choosing the appropriate sizes and number of clusters is essential for the performance of the network lifetime. If the cluster’s radius is too large, it will host many nodes and energy is wasted due to inter-cluster collisions. On the other hand, if the radius is too small, a large number of clusters is needed to cover the observation area, many of them having a large distance to the sink.
In order to enforce standardization among sensor devices, the ZigBee Alliance [5] was formed in 2002 as an association of companies to create a low-cost and low-power transmission standard for wireless personal area networks (WPAN). The ZigBee specification defines the communication on the network layer and above, while the IEEE 802.15.4 standard [6] is adopted for the physical and medium access control (MAC) layers. On MAC layer, access to the channel is controlled with a carrier sense multiple access with collision avoidance (CSMA/CA) algorithm that is especially designed for WPAN. Different network topologies, such as star-shaped and peer-to-peer, are supported. Recently, there has been a growing number of publications dealing with the performance of IEEE 802.15.4, e.g. [7–9]. However, most papers use simulation studies or analytical evaluation of CSMA/CA considering the system to operate under steady state conditions. Our focus in this paper is on an application in which the transmission instants of each node are synchronized, i.e., all nodes simultaneously initiate their transmission attempt. This specific scenario is highly non-stationary and very harmful for CSMA/CA as shown in [10] for IEEE 802.11 Wireless LAN.

In this paper we present an analytic model of IEEE 802.15.4 CSMA/CA in a cluster of sensor nodes. Our main interest is on the transmission delay and the resulting energy consumption due to the MAC protocol. The purpose of this study is to provide a model for evaluating cluster-based methods. In some clustering models the unrealistic assumption can be found that a node operates with several different MAC protocols. In our paper, a sensor node uses only the ZigBee protocol stack, which we will describe in Section 2. This is followed in Section 3 by the analytical model of the CSMA/CA protocol, from which we derive the delay distribution depending on the number of nodes in a cluster and the message length. In Section 4, we investigate the effects of the parameter settings on the energy consumption. Finally, this work is concluded by a brief outlook on future work.

2 System Description of the Sensor Network

The IEEE 802.15.4 standard aims at low rate wireless personal area networks. Therefore, the requirements on operation and power consumption differ from that of other CSMA/CA-based MAC protocols, e.g. IEEE 802.11 for WLAN.

2.1 The IEEE 802.15.4 MAC Protocol

The MAC protocol can operate in two modes: beacon-enabled or beacon-less mode. Both modes use discrete time slots. In the beacon-enabled mode, superframes consisting of 16 slots are used. The first part of the superframe is the contention access period in which CSMA/CA is employed. Up to 7 guaranteed time slots at the end of the superframe may be used for the contention free period that are dedicated to nodes requiring low latency. The acknowledgment of packets is not strictly required and unlike WLAN an RTS/CTS mechanism is not used. In the following we will assume the beacon-less mode with CSMA/CA operating according to the algorithm given in Fig. 1.

The time axis is discretized into backoff units, each with duration of 20 symbol periods. We consider the network to operate in the 2.4 GHz frequency band with a symbol rate of 62.5 ksym/s, i.e., a backoff unit has a length of 80 bits. The variable \( BE \) denotes the backoff
Fig. 1. Flow chart of the CSMA/CA mechanism

$\text{NB} = 0, BE = BE_{\text{min}}$

delay for random $2^{BE-1}$ unit backoff periods

perform CCA

channel free?

yes

$\text{NB} = \text{NB}+1$

$BE = \min(BE+1, BE_{\text{max}})$

no

no

$\text{NB} > M ?$

yes

failure

success

failure

success

$\text{exponent}$ that is increased from $BE_{\text{min}}$ until $BE_{\text{max}}$. The variable $\text{NB}$ characterizes the number of backoffs and is initialized with $\text{NB} = 0$ at the beginning of each round. Default values given in [6] are $BE_{\text{min}} = 3$, $BE_{\text{max}} = 5$ and the maximal number of backoffs is $M = 4$. If the transmission attempt has not been successful until the $M$-th backoff, it will be aborted. Note that the 0-th backoff is always performed. The clear channel assessment (CCA) is a physical layer primitive to check if the channel is busy. We incorporate the time required by CCA in our model by assuming that the transmission can start earliest at the next time slot after the attempt.

2.2 Clustering in Sensor Networks

In order to simplify the electronic circuitry of the sensor nodes, they only gather the information and forward the data to the sink where the processing is done. If all nodes transmit directly to the sink, the system would not be scalable as the many-to-one transmission can consist of hundreds or thousands of nodes. Therefore, clustering methods have been proposed, in which the nodes within a cluster send the data to a designated node, called cluster head. It collects the data locally from the other cluster members and transmits the aggregated data either directly or via multi-hop transmission to the sink, see Fig. 2. Furthermore, if the data packets are locally correlated, they can be compressed to shorter messages [11] at the cluster heads. Since the cluster heads spend more energy than the other cluster members, their role is rotated among all nodes in order to equalize energy consumption.

3 Analytical CSMA/CA Model

In the following, we study the case where all sensors transmit their data in a synchronized manner, which is regarded as the worst case scenario for CSMA/CA. Similarly to [10] our analysis consists of two steps: \(i\) the derivation of the probabilities that each user
attempts to access the channel, and (ii) the computation of the transmission delay. Our focus lies on the impact of two specific input parameters, the number of nodes \( N \) in a cluster and the message length \( L \) discretized in backoff units.

### 3.1 Derivation of the Attempt Probabilities

Let us assume an arbitrary node in a cluster of \( N \) nodes in total. The *backoff duration* \( B_k \) at the \( k \)-th backoff is chosen uniformly between 0 and \( W_k - 1 \) and the total number of backoffs is limited by \( M \).

\[
W_k = 2^{\min(BE_{min} + k, BE_{max})} \quad 0 \leq k \leq M
\]  

\( B_k \) is a uniform random variable and its distribution is given by Eqn. (2). The *total attempt duration* \( D_k \) at the \( k \)-th backoff consists of the sum of all \( B_j, j \leq k \), given in Eqn. (3).

\[
b_k(t) = \Pr[B_k = t] = \frac{1}{W_k} \quad (2)
\]

\[
d_k(t) = \Pr[B_0 + B_1 + \cdots + B_k = t] = b_0(t) \oplus b_1(t) \oplus \cdots \oplus b_k(t) \quad (3)
\]

The probabilities in Eqn. (2) and (3) are defined for \( 0 \leq t \leq t_k = \sum_{j=0}^{k} W_j \) and “\( \oplus \)” denotes the discrete convolution operator.

The number of backoffs prior to a successful transmission is expressed by the random variable \( Q \). Let \( \xi \) be the probability that the channel is found busy while attempting transmission. After comparison with simulation results, we found that approximating \( q_k \) as pmf of \( Q \) with a binomial distribution yielded better results than a using a geometric distribution. The approximation for \( \xi \) is given by the term in Eqn. (4) where \( E[W] \) is the average over all backoff window sizes.

\[
q_k = \binom{M}{k} \xi^k (1 - \xi)^{M-k} \quad \xi = \min\left\{1, \frac{L(N-1)}{E[W]}\right\} \quad (4)
\]

The *attempt probability* \( a(t) \) for a node attempting a transmission at time slot \( t \) is given by the sum over all \( d_k(t) \). Furthermore, the probability of aborting the transmission attempt is defined as \( \eta(t) = d_M(t) \xi \).

\[
a(t) = \sum_{k=0}^{M} d_k(t) \quad (5)
\]
The attempt probability for \( N = 10, L = 2 \) and different values of \( M \) is shown in Fig. 3(a). The curves all show the same distinct sawtooth behavior reported in [10]. However, due to the limited number of \( M \), all curves decrease to zero when \( t \) approaches \( t_M \). Obviously, the attempt probability increases for large \( M \).

Note that the attempt probability is closely related to the delay that each cluster member encounters in a round when it transmits its data to the cluster head. To derive the distribution of the random variable node delay \( T_{\text{node}} \), we must weight the sum over the \( d_k \) with the backoff probabilities \( q_k \). While the attempt probabilities are simply a time series of probabilities, \( \tau_{\text{node}}(t) \) represents a probability distribution where the sum must be normalized to 1. It can be seen in Fig. 3(b) that the analytical distribution matches well with the results from simulation despite the rough approximation of \( \xi \).

\[
\tau_{\text{node}}(t) = \sum_{k=0}^{M} d_k(t) q_k
\] (6)

### 3.2 Calculation of the Total Transmission Delay

After having characterized the behavior of each individual sensor node, the next step consists of taking the influence from the other cluster members into account. Each node starts its transmission process at the synchronization time instant \( t = 0 \) and performs the backoff process as described in Section 2.1. When the node has successfully transmitted the message to the cluster head, it remains silent until the next synchronization instant. We assume that the inter-synchronization interval is large enough to be neglected here.

The total transmission delay \( T_{\text{head}} \) is the time that the cluster head is required to sense the channel in a round until all cluster members have successfully transmitted or aborted their attempt. The pmf of the random variable \( T_{\text{head}} \) will be given by \( \tau_{\text{head}}(t) \).

We assume the following state space as illustrated in Fig. 4. It forms a pure “death” process as no transitions to higher states are made. The states and their corresponding probabilities \( x_{i,j} \) are denoted by the number of nodes \( i \) that have not yet completed their transmission attempt and the number of slots \( j \) it is currently transmitting. Once the absorbing state 0 is reached, the data transmission cycle is completed.
All transition probabilities are time-dependent and are formulated with the help of the attempt probability in Eqn. (5). A successful transmission is described by \( s_i(t) \) when only a single of the remaining \( i \) nodes can successfully access the free channel at time \( t \). The probability for remaining in state \( x_{i,0} \) is given in Eqn. (7) as \( w_i(t) \) when none of the \( i \) remaining nodes accesses the channel. Note that we define \( w_0(t) = 1 \) as a special case. The remaining probability constitutes the probability of collision \( c_i(t) \). Of all the nodes involved in a collision, some will abort their transmission due to reaching the maximum number of backoffs. As there can be multiple simultaneous failures, we model these transitions with a binomial distribution for \( 0 \leq i \leq k \); see Eqn. (8).

\[
\begin{align*}
  s_i(t) &= i a(t) (1 - a(t))^{i-1} \\
  w_i(t) &= (1 - a(t))^i \\
  c_i(t) &= 1 - s_i(t) - w_i(t) \\
  f_i^{(k)}(t) &= c_i(t) \binom{i}{k} \eta^k (1 - \eta)^{i-k}
\end{align*}
\] (7) (8)

The attempt probabilities are time-dependent, therefore, we must perform a non-stationary analysis technique by using the simple power method. We start with an initial probability vector \( x(0) \) and iteratively multiply the state vector

\[
x(t) = [x_{0,0}(t), x_{1,L}(t), \ldots, x_{1,1}(t), x_{1,0}(t), \ldots x_{N,L}(t), \ldots, x_{N,1}(t), x_{N,0}(t)]
\]

with the time-dependent transition matrix \( P(t) \) until \( t = t_M \) is reached.

\[
x(t + 1) = x(t) P(t)
\] (9)

We use \( x(0) = [0, \ldots, 0, 1] \) as initial probability vector, since at time \( t = 0 \) there will be \( N \) nodes in the system with probability 1. The resulting component values \( x_{0,0}(t) \) of the vector \( x(t) \) constitute the cumulative delay probabilities of \( \tau_{head}(t) \). Examples of the distribution are illustrated in Fig. 5(a). The pale colored lines show the distributions obtained from simulations. It can be seen that the boldly colored results from the analysis match well with the simulation values.

### 3.3 Probability of Success

We are interested in the probability of a transmission attempt being successful until \( t = t_M \). Since our Markov model does not distinguish between successful and aborted
attempts, we obtain the number of successful nodes $S_N$ after the round from a modified state space of Fig. 4 where we only distinguish between transitions for successful and unsuccessful attempts. From the state probabilities $x_{i,j}(t_M)$ we then derive the success probability $\varphi_N$. The average number of successful transmissions and success probability are given in Eqn. (10).

$$S_N = \sum_{i=0}^{N} \left[ (N - i) \sum_{j=0}^{L} x_{i,j}(t_M) \right]$$

$$\varphi_N = \frac{S_N}{N} \quad (10)$$

The number of successful nodes is shown in Fig. 5(b) over the total number of nodes. We varied the maximum backoff $M$ in this experiment. It can be clearly seen that increasing the number of nodes leads to a point after which the performance decreases due to collisions. Further, the figure justifies the use of clustering methods from MAC viewpoint and shows that the best operating range is up to approximately 20 nodes. A large $M$ increases the capacity of the cluster, however, the value $M = 10$ used here is only a hypothetical value, as the standard in [6] specifies a limitation of $M \leq 5$.

### 3.4 Approximation of Number of Backoffs

In Eqn. (4) we have used an approximation for the last backoff until success. For the evaluation of the energy consumption in the next section, the average number of backoff attempts is needed. Thus, when we have $N$ nodes, the unsuccessful nodes experience the total number of $M$ backoffs, whereas for the successful $S_N$ the average number of backoffs is given by the expectation of random variable $Q$. Thus, we have as approximation of the average number of backoffs the term $C_N$ in Eqn. (11).

$$C_N = M \left[ 1 - \varphi_N (1 - \xi) \right]. \quad (11)$$

### 4 Evaluation of Energy Consumption

For the evaluation of the energy consumption we consider the distribution of node positions according to a homogeneous spatial Poisson process. The distribution is charac-
characterized by the density $\lambda$ which describes the average number of nodes in a unit area size.

### 4.1 Basic Energy Consumption Model

We adopt the energy consumption model given in [3] for transmitting and receiving data with length $l$ bits for a transmitter and receiver separation of $x$.

$$
E_{Tx}(l, x) = l \left( E_{elec} + \varepsilon_{fs} x^2 \right)
$$

$$
E_{Rx}(l) = l E_{elec}
$$

(12)

The electronics energy $E_{elec}$ is 50 nJ/bit and $\varepsilon_{fs} = 10$ pJ/bit/m$^2$ is the energy for the transmitter amplification in free space. Aggregating data messages consumes $E_{fuse} = 5$ nJ/bit/signal/bit and the energy for sensing requires $E_{sense} = E_{elec}$.

### 4.2 Energy Consumption per Cluster

The total energy consumption per round in a sensor node cluster usually consists of four parts: (i) the transmission energy of the data from the cluster members to the cluster head, (ii) the reception energy of these data messages at the cluster head, (iii) the energy for fusing and aggregating the received data, and finally (iv) the transmission energy of the aggregated data from the cluster head to the sink/next hop. In this paper we only focus on the effects within a single generic cluster and only roughly take into account the steps (iii) and (iv) as they depend on the specific application and the method of how the clusters are generated. However, it should be remarked that this part greatly influences the overall performance and the model in this paper can be regarded as a tool to analyze each individual case when dealing with a specific clustering method.

In a cluster with $N$ nodes, we sort the nodes by their distance $R_n$ to the cluster head which is given by the Erlang probability density function and its average $E[R_n]$ is expressed with the Euler gamma function in Eqn. (13).

$$
r_n(x) = \frac{2(\lambda \pi x^2)^n}{x (n-1)!} e^{-\lambda \pi x^2} \quad \Rightarrow \quad E[R_n] = \frac{\Gamma(n + \frac{1}{2})}{\sqrt{\lambda \pi} \Gamma(n)}
$$

(13)

The average energy for the $n$-th node successfully transmitting a message of length $l$ bits to the cluster head is $E_{node}$. For evaluation of the transmission/reception energy, we consider the success probability given by $\varphi_N$, the energy consumption for the unsuccessful nodes only consists of the energy wasted while sensing.

$$
E_{node}(n) = C_N E_{sense} + l \varphi_N \left( E_{elec} + \varepsilon_{fs} E[R_n]^2 \right)
$$

(14)

The cluster heads must sense the channel until all nodes have either successfully completed the round or aborted their attempt. They receive $S_N$ messages, which are aggregated to a single message and then transmitted to the sink. We assume that the cluster head is located at distance $x$ from the sink and that no compression takes place, i.e., the aggregated message has a length of $l S_N$. The average energy expended at a cluster head
per round is denoted as \( E_{\text{head}} \) and the total energy consumption in the cluster is given by \( E_{\text{cluster}} \) in Eqn. (16).

\[
E_{\text{head}}(N, x) = E[T_{\text{head}}] E_{\text{sense}} + l \left( (S_N + 1) \left[ E_{\text{fuse}} + \varepsilon_{fs} x^2 \right] + (2S_N + 1) E_{\text{elec}} \right) \tag{15}
\]

\[
E_{\text{cluster}}(N, x) = E_{\text{head}}(N, x) + \sum_{n=1}^{N} E_{\text{node}}(n) \tag{16}
\]

The energy consumption normalized per node and packet length is depicted in Fig. 6(a) for the values \( \lambda = 0.01, M = 4, \) and \( x = 50 \). It seems that increasing the number of nodes or the message length reduces the energy consumption. However, from Fig. 6(b) we could see that large \( N \) or \( L \) tend to have a small number of successful nodes. The monotonous decrease in Fig. 6(a) is caused by many node failures and few actual transmissions, so energy is only consumed by sensing. In Fig. 6(b) we mapped the success probability to the dissipated energy. Energy consumption is minimal, when the success probability is high, however, this only occurs for small \( N \). The most energy is consumed in the mid-region where we have the highest absolute number of successful nodes. The optimal operating point of the sensor network should therefore lie somewhere to the right of the maximum point, depending on the desired success probability.

5 Conclusion and Outlook

In this paper we modeled the CSMA/CA algorithm in IEEE 802.15.4 and performed a non-stationary analysis for synchronized transmission attempts. We showed that using the default values given in [6] provides too short backoff window sizes causing many collisions when the number of nodes is large.

With our model we derived the distribution of the total delay for a cluster with \( N \) nodes and the time each individual node needs for its transmission attempt. Based on these distributions, we estimated the average energy consumption for performing a single round of data gathering in one cluster. The results showed that it is indeed very hard to determine a most energy-efficient cluster size by trading off success probability and energy consumption.
However, to give detailed results about the energy consumption of a whole sensor network, a specific clustering method and the costs of its energy overhead [12] need to be analyzed in conjunction with the MAC protocol. In the future we wish to investigate these effects and determine the optimal cluster size when the cluster heads employ direct or multi-hop transmission to the sink. With this paper a framework to facilitate the analysis of clustered sensor networks operating with IEEE 802.15.4 CSMA/CA is provided.

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