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ヒトの脳機能ネットワークに着想を得た高信頼な相互接続ネットワークの設計

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あらまし　近年実用化が進んでいる IoT ネットワーク環境においては、ネットワークを通じて多種多様なサービスを展開するため、IoT ネットワーク同士の相互接続が必須となっている。さらに、交通、航空、電力など、主要な社会インフラもネットワーク化が進み、それらの高度利用を目的として相互接続が進められている。しかし他方では、ネットワークの相互接続は、特に信頼性の観点から相互依存の状態を引き起こし、単一のネットワーク内の障害が他のネットワークへと波及するという問題も認識されている。そこで本研究では、脳機能モジュール間の相互依存性を考慮した、脳ネットワークにおけるインフルエンサーの特定手法に着目した。相互依存性の存在する相互接続ネットワークのネットワーク間接続構造を様々に変化させ、インフルエンサーを評価することにより、インフルエンサーの影響力と配置の一元化及び分散化が可能であることを示した。同時に、相互依存性の存在しない相互接続ネットワークと同等の頑健性を実現可能であることを示した。

キーワード　相互接続ネットワーク, 脳機能ネットワーク, Internet of Things, インフルエンサー

Reliable Architecture for Network of Networks with Inspiration from Brain Networks

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Abstract　The Internet is rapidly developing toward the next generation of the Internet of Things (IoT), which accelerates the emergence of interconnected network architectures even further. In the context of IoT, not only the Internet services becomes sophisticated and diversified, the existing social infrastructure (e.g., transportation, electricity) are expected to be connected and dependent on the Internet. It is pointed that due to the inter-dependency between networks, a partial malfunction in a network can propagate to other interconnected networks. In this study, as an inspiration to solve this issue, we focus on a method to detect influencers in an inter-dependent networks of the human brain. By configuring connectivity patterns between subnetworks of an interconnected network, where inter–modular dependency exist, our evaluation shows the feasibility of centralization and distribution of influencers regarding its influence and deployment. At the same time, our results showed that the deterioration of the robustness caused by inter-dependency can be reduced as that of interconnected networks where no inter-modular dependency exists.

Key words　Network of Networks, Interconnected Networks, Brain Networks, Internet of Things, Influencer

1. Introduction

Information networks are characterized by rapid growth and increased complexity and has been developing toward the next generation of the Internet of Things (IoT). In the context of IoT, not only the Internet services becomes sophisticated and diversified, the existing social infrastructure (e.g., transportation, electricity) or other vital services for everyday life are expected to be connected and dependent on the Internet [1, 2]. These types of interdependent systems are often referred to as *Network of Networks* (NoN). The development toward IoT permits connecting various heterogeneous services in everyday life via the Internet protocol, which results in the acceleration of the emergence of NoN architectures even further.

Virtualized networks is considered as one example of interdependent network systems. In the IoT scenario, virtualization of wireless sensor networks (WSN) has been widely studied [3,4]. A virtualized WSN consists of two network layers: infrastructure providers deploy physical network resources and form infrastructure layer, and multiple service layers are virtually constructed by combining the physical resources on the infrastructure layer. Thus, there exists inextricable interdependency via the infrastructure layer. The service layers are also expected to functionally cooperate to provide further complex and sophisticated services. Another example of interdependent network system would be the functionally interconnected network in smart cities [5]. Toward the future IoT society, the number of IoT devices and the types of services on the Internet have been showing explosive increase. Smart cities intelligently integrate information collected via the IoT devices to improving services in healthcare, surveillance, infrastructure, public utilities, etc., resulting in the realization of smart homes or smart grids. For instance, smart homes could contain air conditioning systems capturing temperature, humidity, and circulation from IoT devices in order to provide best services in response to various requirements. Beside the new situations we can foresee at the moment, also the number of automated and interconnected service systems operating over the IoT infrastructure is expected to drastically increase.

While welcoming the future convenient and sophisticated technological society, there exists a problem behind the collaboration of those different service networks. It is pointed that due to the inter-dependency between networks, a partial malfunction in a network such as device failure or computer-virus infection can propagate to other interconnected networks. Therefore, an appropriate strategy to design interconnected networks that can deal with the above-mentioned problem, which has not been addressed so far.

In this study, as an inspiration to solve this issue, we focus on a previous study in neuroscience field. Morone et al. [6, 7] has proposed a method to model interdependency of modularly connected brain networks and detect influencers, i.e. a set of influential nodes from a topological viewpoint, in the brain networks. The brain network is composed of a number of modules, which provides a specified function (e.g. visual processing) alone, but also provides more complex and sophisticated function (e.g. cognition) in collaboration with other modules. While connections within modules just pass signals over nodes, i.e. neurons, around the modules, Connections between modules control the cooperation of different functions. Hence, a processing halt in a module cause malfunction in other modules. In the light of similarity between information networks and brain networks as interdependent network systems, we apply Morone's method to design influencers in interconnected networks. Not only for detecting influencers or comparing interdependent and non-interdependent systems, we do also aim to configure influence and location of influencers in interconnected networks by changing connectivity between networks.

The purpose of this study is to reveal the way to design interconnected networks that are robust against network malfunction caused by computer-virus or physical disasters, taking into account the inter-modular dependency between subnetworks. Therefore, we first focus on a method to detect influencers in an inter-dependent networks, the human brain networks. We apply this method to reveal the appearance pattern of influencers in interconnected networks with various topological shape, and also to reveal the difference between the networks where inter-dependency are considered or not. Second, by focusing on the assortativity between networks, we propose a method to configure the intra-modular connectivities. Our proposed method helps to know how we can control the influence and location of influencers in interconnected networks. It also reveals how we can compensate the degradation on robustness caused by the inter-dependency between subnetworks. These results will guide the future studies on the construction of interconnected networks with high reliability.

2. Related Works

2.1 Models of *Network of Networks*

As shown in Table 1, in the Brain NoN model, four different node-states are defined using three variables. The variable n_i refers to the existence state of node i, the variable σ_i refers to the local effectiveness of node i, and the variable ρ_i refers to the global effectiveness of node i. The existence state is considered given, and local/global effectiveness of a node depends on the existence state of itself and its neighbors. The definition for σ_i is given as follows:

$$\sigma_i = n_i\left[1 - \prod_{j\in\mathcal{F}(i)} (1 - n_j)\right], \tag{1}$$

where $\mathcal{F}(i)$ denotes the set of nodes connected to node i via inter-modular links. Note that $\mathcal{F}(i)$ takes ϕ when the node i has no inter-modular links. That is, the condition for node i to become active

Table 1 Definition for node state

Sign	State	n_i	σ_i	ρ_i
○	removed	0	0	0
●	exist	1	0	0
◉	locally effective	1	1	0
● (red)	globally effective	1	1	1

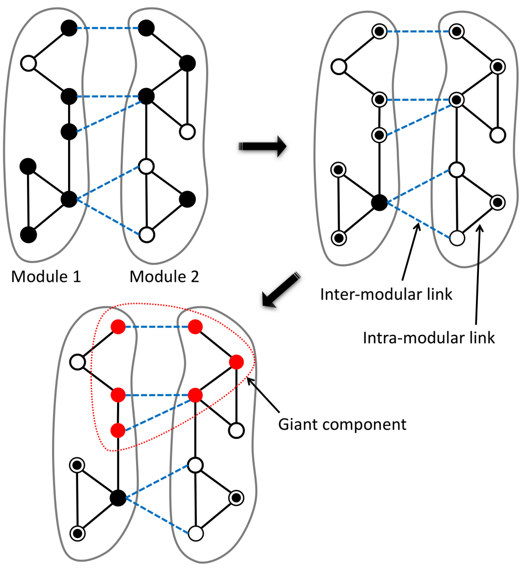


Figure 1 Example of node-state transition in Brain NoN model [6]

differs depending on whether it has inter-modular links. If it has one or more inter-modular links (i) node i have to be INPUT and (ii) at least one node connected to node i via inter-modular links have to be INPUT. Meanwhile, if it has no inter-modular links, the condition is just (i) node i have to be INPUT. This variable σ_i implies that a node that has inter-modular links cannot be useful without support from other adjacent modules since the service provided on this network of networks is interdependent.

In the Brain NoN model, a node is regarded as globally effective only when it belongs to the giant component of an interconnected network. Whether a node is included in the giant component is calculated through a percolation analysis using a message passing method. On the message passing, each node first calculates the probability whether the node itself belongs to the giant component based on the information from adjacent nodes. Then, the node passes the resultant probability to all its adjacent nodes. The probability is defined as follows:

$$\rho_i = \sigma_i \left[1 - \prod_{k \in \mathcal{S}(i)} (1 - \rho_{k \to i}) \right] \tag{2}$$

where $\mathcal{S}(i)$ denotes the set of all adjacent nodes of node i, and $\rho_{k\to i}$ denotes the information from the adjacent nodes, i.e. the probability for them to be included in the giant component. Starting with a set of random values for φ and ρ, nodes continue passing information until the values converge. Then, the converged probability is regarded as the global effectiveness.

The signs in Table 1 corresponds to the node state in Fig. 2, which shows an example of state transition in a Network of 2 Networks (2-NoN). Black lines correspond to intra-modular links and blue ones to inter-modular links. The set of nodes circled by a red dotted line corresponds to the giant component detected in the perforation procedure described above.

2. 2 Identifying Influencers in Network of Networks

This study deals with identification of influencers to enhance the robustness and communication efficiency of interconnected networks with inter-modular dependency. Influencers are a set of vital nodes in terms of the network connectivity or communication efficiency. Identification of those vital nodes that maximizes the influence over a network is known as NP-hard problem [8], and there exist many heuristic methods have been proposed so far [9], such as betweenness centrality or page-rank. This study focus on one of recent works [7] which proposed the Collective Influence (CI) algorithm to identify influential nodes. Not only does the CI algorithm show superior performances for the identification of influential nodes in comparison with other conventional methods, it is also optimized for interdependent NoN.

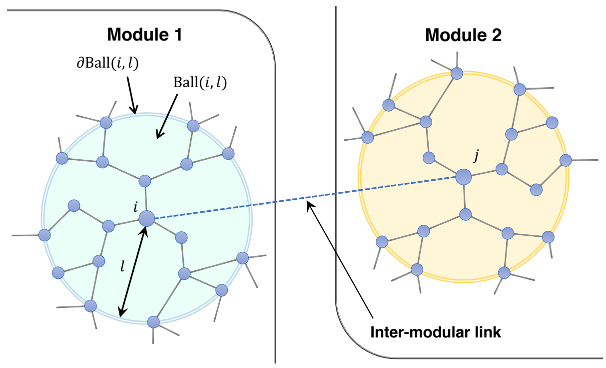


Figure 2 Description of Collective Influence

CI of node i represents its influence on other nodes in the same network centered around node i. CI of node i for a single network is defined as follows:

$$\mathrm{CI}_l(i) = (k_i - 1) \sum_{i' \in \partial \mathrm{Ball}(i,l)} (k_{i'} - 1) \tag{3}$$

where k_i denotes the degree of node i, $\mathrm{Ball}(i,l)$ denotes the set of nodes within l hops centered around node i, and $\partial\mathrm{Ball}(i,l)$ denotes the set of nodes on the edges of $\mathrm{Ball}(i,l)$.

Morone et al. [7] also define the CI algorithm for inter-dependent modular networks as follows:

$$\begin{aligned}\mathrm{CI}_{l\geq 1}(i) &= k'_i \sum_{i' \in \partial\mathrm{Ball}(i,l)} k'_{i'} \\ &+ \sum_{j \in \mathcal{F}(i): k_j^{out}=1} \left[k'_j \sum_{j' \in \partial\mathrm{Ball}(j,l)} k'_{j'} \right]\end{aligned} \tag{4}$$

Note that the variable k considers both links within and between modules. The first term of the equation corresponds to CI of node i itself, and the second term corresponds to the sum of CI of node j which is connected to the node i via inter-modular links. The condition $k_j^{out} = 1$ means that the node j is taken into account if it has only an inter-modular link. This is because if node j has more than one inter-modular links, it is not directly affected by the state of node i.

3. Method for Constructing Interconnected Networks

This study aims to reveal the relation of the following three elements: the performance of interconnected networks, the interdependency between subnetworks, and the intra-modular connectivity. In this study, we focus on the assortativity for the intra-modular connectivity, and thus, we describe how to construct an interconnected network topology with configuration on inter-modular assortativity for the evaluation.

3.1 Assortativity between Networks

Newman was the first person who proposed measuring the assortativity of a network with the assortativity coefficient [10]. Then, a following research introduced universal assortativity coefficient to define the assortativity between networks [11]. This coefficient reflects the contribution of an individual edge's to the assortativity of the entire network, i.e. Newman's assortativity [10], The assortativity coefficient is calculated from the remaining degree distribution $q(k)$ defined as follows:

$$q(k) = \frac{(k+1)p(k+1)}{\sum_j jp(j)}, \tag{5}$$

where $p(k)$ denotes the probability that a randomly selected node has nodal degree of k.

Then, the universal assortativity coefficient ρ_l on a link l can be introduced given $q(k)$. The definition of the universal assortativity of link l is as follows:

$$\rho_l = \frac{(j-U_q)(k-U_q)}{L\sigma_q^2}, \tag{6}$$

where j and k denote the remaining degrees of the two endpoints of link l, which have the same expected value of the remaining degree $U_q = \sum_j jq(j)$. The term L denotes the number of links in the whole network, and the term $\sigma_q^2 = \sum_l j^2 q(j) - \left(\sum_k kq(k)\right)^2$ denotes the variance of the remaining degree distribution $q(k)$. As seen in Fig. 3, when $\rho_l > 0$, the link is called an assortative link; otherwise when $\rho_l < 0$, a disassortative link. A link with $\rho = 0$ has no correlation.

Then, we use the term $\rho = \sum_{l \in S} \rho_l$, which reflects the sum of the assortativity of all inter-modular links, for measuring the assortativity between networks.

3.2 Construction of Interconnected Networks

First, two subnetworks are prepared beforehand, that composes an interconnected network. We deploy a set of N nodes for each subnetwork, and connect L_{intra} links among the nodes based on

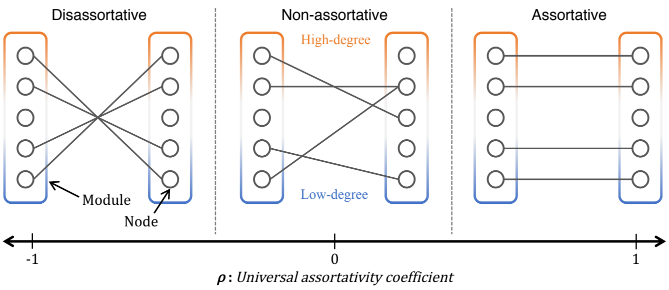


Figure 3 Connectivity between modules and the universal assortativity coefficient

a certain network models, which are commonly used in graph-theoretical evaluation. In this study, connectivity pattern for inside subnetworks are considered as given, and thus, the description for the network model is described in the following section.

Then, L_{inter} links are assigned between the two subnetworks so that the connectivity achieves a specified value for inter-modular assortativity of the interconnected networks. In order to obtain a appropriate connectivity pattern for the assortativity, inter-modular links are rewired repeatedly. The rewiring process is conducted stochastically with the connectivity pattern determined by the following procedure:

(1) L_{inter} edges are randomly assigned between the two subnetworks. Note that no endpoint node have multiple inter-modular links.

(2) The assortativity between the modules ρ is calculated. If ρ matches the targeted value, the procedure is finished. Otherwise, the following steps are repeated.

(3) An existing link between the modules, whose assortativity is farthest away from the target value, is deleted. If the current assortativity ρ is higher than the targeted value, the most assortative inter-modular link is selected, and vice versa.

(4) A new link is created on two nodes in two different modules. The pair of nodes are randomly selected under the condition that the new link can move the assortativity ρ closer to the target value. Then, go back to Step 2.

The resultant topology of interconnected networks are used for the evaluation.

4. Evaluation

In this section, we evaluate the performance of interconnected networks. By this evaluation, we aim is to reveal topological characteristics of influencers in interconnected networks, comparing the cases when inter-modular dependency exists and not. We also reveal the way to enhance the robustness and efficiency by configuring the inter-modular assortativity in interconnected networks.

4.1 Evaluation Environment

First, we describe the settings for the simulation evaluation. As explained in Sec. 3., we construct topologies of interconnected networks composed of two subnetworks. Each network contains $N = 100$ nodes, and forms a topology with $L_{intra} = 300$ links based on the network models explained in Sec. 4.1.1. After creating subnetworks, we assign L_{inter} inter-modular links between the two subnetwork so that the connectivity pattern achieves a specified value of ρ. We set the value for L_{inter} equal to N and let every node have just an inter-modular link to simplify the problem. Although the range of ρ ideally takes -1 to 1, it is much more limited because of the topological shape of each subnetwork. To reveal the deference between interconnected networks with inter-modular dependency and without, see Sec. 4.1.2. The metrics for evaluating influencers are described in Sec. 4.1.3

4. 1. 1 Intra-Modular Connectivity

a) *Erdös-Rényi* (ER) Model

The ER model is a class of random network models [12]. The degree distribution of the ER model follows a Gaussian distribution that is similar to the distribution observed in information networks. For the network construction, we repeat randomly choosing a pair of nodes and connecting them until the number of generated intra-modular links reaches L_{intra}.

b) *Barabáshi-Albert* (BA) Model

The second type of networks corresponds to the BA model [13], which is one of the most well-studied complex network models. The BA model is characterized by that there exists extremely high-degree nodes (i.e. hub nodes) and a core cluster of hub nodes (i.e. rich-club) and that its degree distribution follows a power-law.

For the topology construction, we first pick a small set of nodes to create an initial full-mesh topology. Then, we repeatedly add a new node to the initial topology, and at each time we probabilistically choose m other nodes from the existing topology for generating a link between those nodes and the new node. At this time, the probability that node i is chosen is given by $p_i = k_i/\Sigma_j k_j$, where k_i is the degree of the node i. Hence, m is chosen so that almost the same number of intra-modular links (L_{intra}) are finally generated.

4. 1. 2 Inter-Modular Dependency

As mentioned in Sec. 2. 1, we apply the rule of node-state transition in the Brain NoN model to the generated interconnected networks. In addition, we also define Pure NoN, a basic model that does not consider the interdependence between subnetworks. For the Pure NoN, we replace Eq. 2 with $\sigma_i = n_i$ and inter-modular links works just the same as the intra-modular links. The comparison of Brain NoN and Pure NoN let us know what we have to consider on the construction of inter-dependent networks.

4. 1. 3 Evaluation Metrics

We evaluate the performance based on hop length among nodes.

a) Average Hop Length of Remaining Effective Nodes

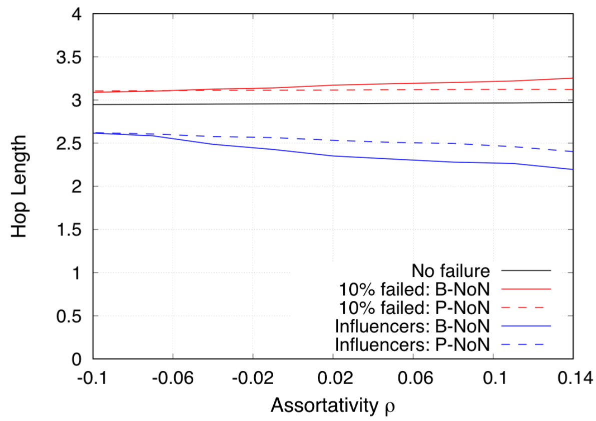
In order to measure the effect of influencers, we remove influential nodes using CI algorithm on an interconnected network and see the change of average hop length (AHL) of remaining networks other than the failed parts. Here, AHL is defined as the average of the number of hops among all the pairs of nodes. It should be noted that only nodes with $\rho = 1$ are considered in the calculation of AHL. Since we focus on the most influential nodes of influencers, the hop count is measured when 10% of nodes are removed.

b) Average Hop Length of Influencers

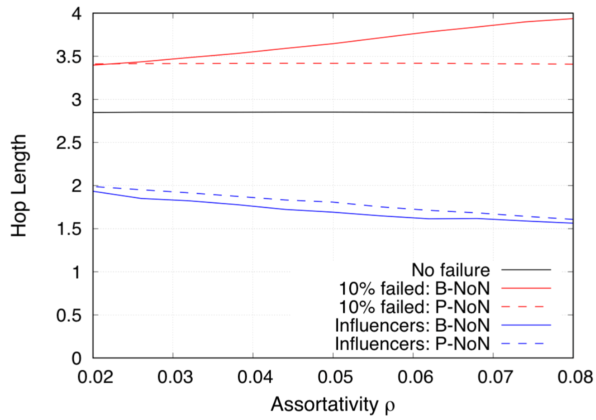
We also check the appearance patterns of influencers by focusing of the hop length among influencers. After detecting influencers as 10% of all nodes after generating network failure, we count hop count of all the pairs of influencers based on the original topology of interconnected networks. By doing this, we confirm how sparsely the influencers are located.

4. 2 Evaluation Results

Figures 4(a) and 4(b) shows the evaluation results for the ER and BA model, respectively. The x-axis corresponds to the assortativity between subnetworks, and the y-axis corresponds to the hop length. As seen in the x-axis, the range of assortativity is quite limited because of the number of inter-modular links and the degree distribution of subnetworks. Black line shows AHL of all the possible pairs of nodes in an original interconnected network topology, red line shows AHL among globally effective nodes when 10% of nodes are removed, and blue line shows AHL among influencers. Each figure is the compilation of the results from 100 computer simulations.



(a) ER model



(b) BA model

Figure 4 Average hop length (AHL) for an interconnected network of no failure (black), AHL of 10%-node failed topology (red), and AHL of influencers (blue)

Regarding the black line, it is interesting that AHL of all nodes do not change even if we configure the inter-modular assortativity between modules. Our previous research showed assortativity between modules affect performance of an interconnected networks, where inter-modular dependency are not considered. This time, however, we assign an inter-modular link to every node in each subnetwork, and thus, average hop length of all nodes converge on the same value.

According to the red lines, when top 10% of influential nodes are removed, the networks of Brain NoN and Pure NoN showed different behavior. Both the lines are shifted toward upper side, because of the lack of influencers, i.e., high-centrality nodes. However, red-dotted line is almost horizontal because the same nodes are removed as influencers regardless of inter-modular assortativity. It is true although the same set of nodes are removed, inter-modular connectivity differs. However, the effect is considered too small to change the performance similarly to the black line. Meanwhile, for the network of Brain NoN, the hop count increases with the inter-modular assortativity. For disassortative networks, even if highly influential node is removed and the other node connected via an inter-modular link is turned down due to the inter-dependency, the latter node is of quite low importance for sustaining connectivity. Whereas for assortative networks, influential node is connected to another influential node, and thus, malfunction on one side becomes quite critical.

As for the blue lines, we observe that influential nodes are basically connected close to each other, and located further closer when assortativity increases. In assortative networks, pairs of influential nodes are connected via inter-modular links, which resulting in the closeness of influencers. The results also show that hop length among influencers slightly decreases when Brain NoN model is applied. This is because a node is more likely to be chosen as influencers if its adjacent node in the other module is also highly influential.

When comparing Figs. 4(a) and 4(b), we can see some differences on the performance. Both the red and blue lines showed larger gaps to the black line. This characteristic can be attributed the degree distribution of the BA model. There exist extremely high degree node in scale-free networks, and thus influencers are closely connected and hop length increased when influencers are removed.

5. Conclusion and Future Work

In this study, we applied a method to detect influencers, i.e. important node on robustness or communication, in interconnected networks that have inter-modular dependency. We also configured the connectivity patterns between subnetworks from the viewpoint of assortativity.

It is conceivable that interdependent networks are more fragile since a partial network failure affects the other connected module. However, our results showed that if inter-modular connectivity corresponds to the disassortative mixing, the deterioration of the robustness can be reduced as that of interconnected networks where no inter-modular dependency exists. At the same time, to put it the other way, if we assortatively connect the subnetworks, unitary management would be much easier since the influencers have higher influence over the interconnected networks. We also confirmed that as the inter-modular assortativity increases components of the influencers are located closer. This implies that assortative mixing between subnetworks help centralization of influencers from topological viewpoint as well, whereas disassortative mixing is preferable to geometrically distribute the influencers. These findings would guide to design reliable interconnected networks that match various environmental requirements in the future IoT context.

However, there remain several issues yet to be tackled to design interconnected and interdependent networks. In the context of information networking, we should take into consideration that different service modules are mutually connected via control plane or operated by the control plane. For example, network virtualization is realized by the combination of virtualized service layer and physical control plane. Social infrastructure services such as transportation or electricity are also connected to and controlled by the Internet. Another untested point is regarding the connectivity within subnetworks. Although we used the ER and BA model, not only the degree distribution, but configuring assortativity on a given degree distribution would help to control the influence and appearance pattern of influencers.

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