Designing Interconnected Networks for Improving Robustness and Efficiency

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***Abstract*—The Internet is rapidly developing toward the next generation of the *Internet of Things* (IoT), which accelerates the emergence of interconnected network architectures even further. However, the way to design interconnected networks that can meet various changes in environment and service demands remains an important issue that has not been appropriately addressed yet. These interconnected networks should be robust to suppress or prevent diffusion of malicious information, whereas they should also be efficient to enhance diffusion of urgent information throughout the entire network. In this study, we propose a *Network of Networks* (NoN) model inspired by the modularly interconnected networks in the brain, and show that the model has two prominent characteristics. First, unintentional information diffusion from one subnetwork to another can be prevented. Second, speed of information diffusion can be controlled even when the diffusion starts from interconnecting links across two subnetworks. For the second point, we further investigate a strategy for influencing the information diffusion speed by appropriately configuring the connectivity within and between subnetworks. Simulation experiments confirm that by adapting the brain-inspired activation rules on the interconnecting links, our model is able to switch from the unhindered and fast information diffusion between subnetworks to the slow case where all interconnecting links are inoperative.**

***Index Terms*—Network of Networks, Brain networks, Internet of Things, assortativity, centrality**

I. INTRODUCTION

Modular interconnected networks, often referred to as *Network of Networks* (NoN), have been frequently observed in many complex systems in biology, society, science and technology, as well as the Internet [1], [2]. In contrast to the other rather static kinds of complex systems, the Internet itself is rapidly developing toward the next generation of the *Internet of Things* (IoT), which permits connecting various types of interconnected devices from everyday life via the Internet protocol and which is expected to accelerate the emergence of modular network architectures even further.

An example of the modular architecture in the future Internet is the functionally interconnected network in smart cities [3]. In the future IoT society, the number of IoT devices connected to the Internet as well as the types of services provided through the Internet are expected to show an explosive and continuous increase. Smart cities automatically collect data from those IoT devices and intelligently integrate them for improving services in healthcare, surveillance, infrastructure, public utilities, etc., resulting in the realization of smart homes or smart grids. For example, smart homes could contain air conditioning systems capturing temperature, humidity, and circulation from IoT devices in order to provide best services in response to a variety of situations. Beside the new situations we can foresee at the moment, also the number of automated and interconnected service systems operating over the IoT infrastructure is expected to drastically increase in future smart cities.

When providing services, these interconnected networks will be required to meet various changes in environment and service demands: robustness to suppress or prevent diffusion of malicious information or efficiency to enhance diffusion of urgent information. However, the best way to design an NoN architecture remains an important issue that has not been fully addressed yet. Therefore, we first focus in this paper on models for NoN that have been studied for modularly interconnected networks [4], [5]. It has been shown that especially biological systems have high robustness against cascading failures. For example, Morone *et al.* [5] proposed an NoN model from the perspective of neuroscience, i.e., networks of neurons in the brain. This NoN model, termed as *Brain NoN* hereafter, considers the characteristics of an activation rule of neural firings in brain networks, which is well-known for its high robustness [6], [7].

Although the activation rule in Brain NoN can be a clue to improve robustness of emerging interconnected Internet services, its application has not been considered so far. Moreover, there are two important questions that need to be answered for interconnected networks regarding their structural connectivity: (i) *how is the connectivity of nodes within modules?* and (ii) *how is the connectivity between modules?* In this study, we attempt to answer those questions from the viewpoint of *influential nodes* and *node correlations*. Influential nodes in networks are defined as the fraction of nodes that have a large influence over the whole network [8]–[12], whereas node correlation [13], [14] is formulated based on the correlation of degrees of two nodes and termed as *assortativity*.

The aim of our work is to design an NoN architecture

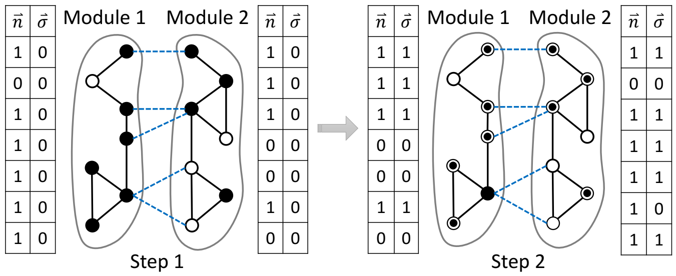


Figure 1. Activation rule in the Brain NoN model [5]

for information networking that meets environmental changes and service demands, which can be summarized as high robustness against malicious information and efficiency for urgent information. For this goal, we first propose an NoN model inspired by Brain NoN that matches situations in information networking with service interdependency. Second, by taking the node influence and node correlations into account, we propose a method to configure the intra- and inter-modular connectivities and evaluate the performance of the NoN. Results from simulation experiments reveal that unlike conventional NoN models without the activation rule on interconnecting links, our proposed NoN model is able to switch from the unhindered and fast information diffusion to the slow case where all interconnecting links are inoperative by configuring its connectivity.

This paper is organized as follows. Section II describes related works already done in the field of graph theory. Section III introduces an NoN model inspired by the human brain, and Section IV describes the method to configure connectivity of interconnected networks. Section V describes the results from evaluations of interconnected networks. Finally, Section VI provides the conclusion and future work.

II. Related Work

A. Models of Network of Networks

In the Brain NoN model [5], nodes can have three different states: *active, input*, and *no-input*. Each node can be active only when its own and its neighbors' input satisfy certain conditions. These three states of node i are determined by two variables, input variable n and activation variable σ, as follows.

◉ : active $(n_i = 1, \sigma_i = 1)$
● : input $(n_i = 1, \sigma_i = 0)$
○ : no-input $(n_i = 0, \sigma_i = 0)$

The patterns of each circle represent nodal states corresponding to Fig. 1, which shows an example of the state transition in a Network of 2 Networks (2-NoN) of the Brain NoN model. The values for the input variable n are assumed as given and they sequentially determine the values for the activation variables σ. Node i can be active only when its own input and the input of at least one node in the other modules exists. The value of σ is defined as

$$\sigma_i = n_i \Big[1 - \prod_{j \in \mathcal{F}(i)} (1 - n_j) \Big], \tag{1}$$

where $\mathcal{F}(i)$ denotes the set of all nodes connected to node i via inter-modular links.

B. Identification of Influence in Networks

Our study also focuses on the vital nodes in order to control acceleration and suppression of information diffusion in interconnected networks. The identification of the set of nodes that maximizes the influence over a network is known as NP-hard problem [8] and several heuristic solutions have been proposed to solve this problem [12].

We focus on [11] which proposed the *Collective Influence* (CI) algorithm to identify influential nodes. CI of node i represents its influence on the other nodes in the same network centered around node i, e.g., betweenness centrality, pagerank, or k-core. The CI algorithm shows superior performances for the identification of influential nodes compared to other methods using conventional centrality measurements by finding the smallest set of nodes that totally collapses the connectivity of the networks. CI of node i is defined as

$$\mathrm{CI}_l(i) = (k_i - 1) \sum_{j \in \partial \mathrm{Ball}(i,l)} (k_j - 1),$$

where k_i denotes the degree of node i, $\mathrm{Ball}(i,l)$ denotes the set of nodes within l hops centered around node i, and $\partial\mathrm{Ball}(i,l)$ denotes the set of nodes on the edges of $\mathrm{Ball}(i,l)$.

C. Universal Assortativity

Assortativity, i.e., the correlation of nodal degrees, was first proposed by Newman with the *assortativity coefficient* [15]. The assortativity coefficient is calculated from the remaining degree distribution $q(k)$ defined as follows:

$$q(k) = \frac{(k+1)p(k+1)}{\sum_j jp(j)},$$

where $p(k)$ denotes the probability that a randomly selected node has nodal degree of k.

Furthermore, the *universal assortativity coefficient* ρ_l on a link l was introduced to analyze the assortativity of any part of a network in [14] and is introduced given $q(k)$. The definition of the universal assortativity of link l is as follows:

$$\rho_l = \frac{(j - U_q)(k - U_q)}{M\sigma_q^2}, \tag{2}$$

where j and k denote the remaining degrees of the two endpoints of link l, which have the same expected value of the remaining degree $U_q = \sum_j jq(j)$. The term M denotes the number of edges in the whole network and the term $\sigma_q^2 = \sum_l j^2 q(j) - \left(\sum_k kq(k)\right)^2$ denotes the variance of the remaining degree distribution $q(k)$. When $\rho_l > 0$, the link is called an *assortative link*; otherwise when $\rho_l < 0$, it is a *disassortative link*.

Table I
MAPPING VARIABLES FROM BRAIN NON TO INFORMATION NETWORKS

variables	Brain NoN	IC NoN
$\sigma = 0$	node is inactive	outer-interface is inactive
$\sigma = 1$	node is active	outer-interface is active

III. INFORMATION DIFFUSION MODEL FOR INTERCONNECTED NETWORKS

Although input to nodes and activation as the result of this input were considered in the Brain NoN model [5], effects of node activation on neighbor nodes have not been considered so far. We expand the activation rule of the Brain NoN model to express the communication flow in interconnected networks.

First, we change the interpretation of the node states in the Brain NoN model to states of node interfaces (network devices) in information communication NoN, called *IC NoN*. In the Brain NoN model, the activation of interconnecting links is coupled with the activation of endpoint nodes of the interconnecting links. In information networks, however, even if one endpoint node is deactivated and thus the interconnecting link is also deactivated, the other endpoint node can maintain its process within the module to which the node belongs.

For this reason, the meaning of the states defined by σ in Brain NoN is reinterpreted as shown in Table. I, where the activation of nodes in Brain NoN now corresponds to the activation of *outer-interfaces*. In this context, the input variable n in Brain NoN represents the input state of information. It should be noted that *inner-interfaces* are always active independent of the value of σ or n. Adding to IC NoN, we note a basic model that does not consider the interdependence between modules as *Pure NoN* in Table I. Pure NoN always diffuses at the maximum speed that the topological connectivity can produce.

Second, in order to express the flow of information, IC NoN adopts the notion of time-scale. In this model, the value of variables n and σ at current time step t is given by the previous states at time step $t-1$. We then introduce a probability function p_t for nodes to decide whether to have input or depend on the states of neighbor nodes. Here, we suppose that each node can pass information with probability δ through active outer- and inner-interfaces whenever they have inputs. Therefore, the probability function $p_t(i)$ for node i to judge whether to have input is written as follows:

$$p_t(i) = 1 - \prod_{j \in \mathcal{S}(i)} (1 - \delta n_j^{t-1}) \times \prod_{k \in \mathcal{F}(i)} (1 - \delta \sigma_i^{t-1} \sigma_k^{t-1} n_k^{t-1}),$$

where $\mathcal{S}(i)$ denotes the set of neighbors of node i within the same module and $\mathcal{F}(i)$ denotes the set of neighbor nodes in the other modules. It should be noted that all inner-interfaces are always active, while outer-interfaces are active only when $\sigma = 1$. An important point this equation expresses is that when $\delta < 0$, node i can behave differently depending on the number of neighboring input nodes: the more input neighbors node i has, the more likely node i will have input.

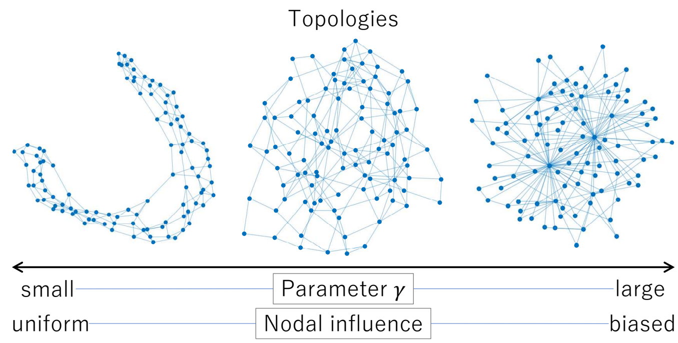


Figure 2. Network topologies with various connectivity

Then, activation state of node i is rewritten based on the rule in Eq. (1) of the Brain NoN model as follows:

$$\sigma_i^t = n_i^t \Big[1 - \prod_{j \in \mathcal{F}(i)} (1 - n_j^t) \Big]. \tag{3}$$

Equation (3) shows that the inter-modular interface of node i becomes active only when node i and at least one neighbor node on an inter-modular link has input.

IV. METHOD FOR CONFIGURING CONNECTIVITY OF INTERCONNECTED NETWORKS

In our strategy, we configure intra- and inter-modular connectivity from the perspectives of node influence mentioned in Section II-B, since node influence and diffusion speed are closely related.

A. *Configuring Connectivity within Subnetworks*

In order to increase/decrease the power of influential nodes in terms of information diffusion speed in a subnetwork, we extend the conventional preferential attachment method and generate topologies by controlling parameter γ. Given a seed network, we add nodes one by one where each new node is connected to m existing nodes. The probability for each link of new node to be connected to an existing node i is defined as follows:

$$p(i) = \frac{k_i^\gamma}{\sum_j k_j^\gamma},$$

where k_i denotes the degree of node i. This process finishes when all N nodes are added to the network. Figure 2 shows the topological structure of networks with changes of the parameter γ. When γ decreases, the topology approaches a uniform degree distribution with average degree $\bar{k} \simeq 2m$ and variance of degree approaches zero, and thus node centrality is also distributed. However, when γ increases, more highly influential nodes emerge and the number of influencers decreases. As a result, topologies will have a node degree distribution following a power-law $p(k) \sim k^{-\delta}$.

B. Configuring Connectivity between Subnetworks

When adding an interconnecting link to an NoN, we consider two points: (i) dependency on the centrality of both endpoint nodes within each subnetwork, and (ii) dependency on the correlation of centrality of both endpoint nodes. All possible pairs of nodes with a certain centrality value can be expressed by changing these two dependencies respectively. Based on this idea, we investigate which nodes should be preferentially selected as endpoint nodes of interconnecting links for achieving an NoN topology with fast/slow information diffusion. In the following, we will formulate each dependency as *Dependency Coefficient* (DC).

1) Coefficient for Node Centrality: First, we define the DC of centrality itself as DC_{cnt}. Here, we consider the dependency on centrality of each endpoint node of interconnecting links independently, DC_{cnt} is simply defined as the sum of centrality of each endpoint node as follows:

$$DC_{cnt}(h, i) = c_h + c_i,$$

where c_h denotes any centrality value of node h within each subnetwork the node belongs to. The values of c_h and c_i, respectively, vary in the range of $[0, 0.5]$: a high value represents high centrality, and vice versa.

2) Coefficient for Correlation of Node Centrality: We measure the correlation of node centrality based on the ideas of universal assortativity as mentioned in Section II-C. The universal assortativity is introduced to measure the correlation of node degree centralities between networks as shown in Eq. 2. Here, the expected value $U_q = \sum_j jq(j)$ is based on the remaining degree. We assume that interconnecting links are generated between two *different* subnetworks independently of the connectivity within each subnetwork. The probability for selecting any node in each subnetwork as an endpoint node is equal. Setting $p(c)$ as any kind of node centrality of a subnetwork, the expected value of the centrality on an endpoint node of an interconnecting link is also expressed as $p(c)$. Therefore, we define another generalized universal assortativity ρ'_l of an interconnecting link l between subnetworks 1 and 2, modifying Eq. (2), as follows

$$\rho'_l = \frac{\left(c_{l_1} - U_{p_1}\right)\left(c_{l_2} - U_{p_1}\right)}{\sigma_{p_1}\sigma_{p_2}}, \tag{4}$$

where c_{l_1} and c_{l_2} denote centralities of endpoint nodes in subnetworks 1 and 2, respectively. U_{p_1} and U_{p_2} denote the expected values of node centrality, defined as $U_p = \sum_j jp(j)$, $\sigma^2_{p_1}$ and $\sigma^2_{p_2}$ denote the variances of node centrality distribution $p(c)$, given as $\sigma^2_p = \sum_l l^2 p(l) - \left(\sum_m mp(m)\right)^2$. Particularly, if subnetworks 1 and 2 have the same node centrality distribution $p(c)$, Eq. (4) can be rewritten as:

$$\rho'_l = \frac{\left(c_{l_1} - U_p\right)\left(c_{l_2} - U_p\right)}{\sigma^2_p}. \tag{5}$$

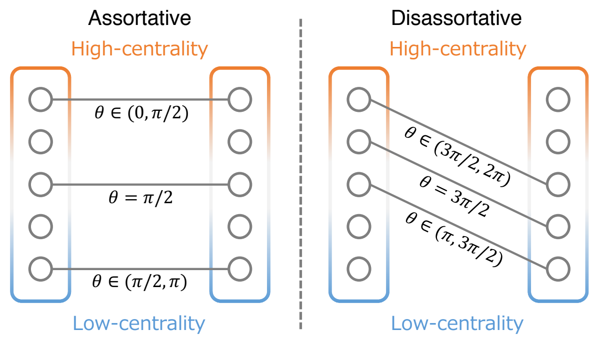


Figure 3. Various patterns of connectivity between subnetworks

Finally, we define DC_{cor} of the correlation of node centralities of the two endpoint nodes by slightly changing the generalized universal assortativity as follows:

$$DC_{cor}(h, i) = \frac{\left(c_h - U_p\right)\left(c_i - U_p\right)}{\sigma^2_p},$$

where h and i are node indices.

3) Coefficient for Varying Connectivity between Subnetworks: To configure the connectivity between subnetworks, we consider two aspects as mentioned above: (i) dependency on centrality of both endpoint nodes, and (ii) dependency on correlation of centrality among the two endpoint nodes. That is, we combine DC_{cnt} and DC_{cor} and express various interconnectivities between subnetworks using DC defined as follows:

$$\begin{aligned} DC(h, i) = \quad & \left[\frac{DC_{cnt}(h, i) - DC^{min}_{cnt}}{\overline{DC}_{cnt} - DC^{min}_{cnt}} + 1\right]^{r\cos\theta} \\ & + \left[\frac{DC_{cor}(h, i) - DC^{min}_{cor}}{\overline{DC}_{cor} - DC^{min}_{cor}} + 1\right]^{r\sin\theta} \end{aligned} \tag{6}$$

where $\theta \in [-1, 1]$, $r \in \{0, 1\}$ with $r = 0$ for random connectivity and $r > 0$ for various connectivity. Each dependency coefficient is normalized so that the effect of both coefficients becomes the same on average. We then add 1 to both coefficients so that the minimum dependency coefficient among all pairs of nodes always stays 1 as a standard value independent of θ.

For $r > 0$ and $\theta \in (0, \pi)$, interconnecting links become assortative; otherwise when $\theta \in (\pi, 2\pi)$, the links become disassortative. When $\theta \in (3\pi/2, \pi/2)$, high centrality nodes tend to be selected as endpoints of interconnecting links, while when $\theta \in (\pi/2, 3\pi/2)$, low centrality nodes are preferred. These cases are depicted in Fig. 3.

V. Simulation Evaluation

A. Simulation Settings

We evaluate the performance of the introduced NoN models by changing their topological connectivity. In particular, we consider the IC NoN model as our proposal and Pure NoN

Table II
PARAMETER SETTINGS

Variables	Values	Description
δ	0.5	information passing probability
γ	[-50,20]	parameter γ for preferential attachment
m	2	parameter m for preferential attachment
r	1	parameter r for connectivity between subnetworks
θ	[0,2π)	parameter θ for connectivity between subnetworks
N	100	number of nodes in each subnetwork
E	25	number of inter-modular links
k_{in}^{max}	25	maximum nodal degree of intra-modular links
k_{out}^{max}	1	maximum nodal degree of inter-modular links

as a reference for comparison. Both models are described in Sect. III.

To conduct the evaluation, we configure the parameter settings on NoN models and topologies according to Table II. We measure the required time-steps for information to diffuse over the entire NoN topologies to evaluate whether the NoN diffuses information quickly or slowly. Two alternatives are considered as origin of the diffusion: (i) the highest loaded inter-modular links, and (ii) randomly selected inter-modular links. Starting the diffusion from an interconnecting link matches both our research objective and the natural behavior of information networking. Although it is the original behavior of the IC NoN model that nodes become empty after passing their information, we designate the source inter-modular link, i.e., the source endpoint nodes, to continuously send the information. This is because the diffusion is following a probabilistic method and it is therefore possible for the diffusion to disappear from the network in the first few iteration steps.

B. Basic Properties of Independent Subnetwork

Before we start with evaluating the information diffusion efficiency, we first investigate the basic properties of each subnetwork of interconnected networks, which will allow for a deeper understanding on how to evaluate interconnected networks in the following section. Figure 4 shows the maximum collective influence (left axis) and the required steps for information diffusion (right axis) in the subnetwork. We can confirm that as the parameter γ increases, the maximum collective influence monotonously grows and the number of required steps decreases. This result implies that the impact of influential nodes can be summarized and distributed by changing the parameter γ. However, we can also find that there is a limitation on the feasible values of maximum collective influence and required steps. We can also find a straightforward tendency of the speed of information diffusion increasing when γ increases.

C. Simulation of Information Diffusion

In this subsection, we investigate the performance of both NoN models, IC NoN and Pure NoN, through simulation of information diffusion. Information diffusion selects influential links as the source of the diffusion based on the average cumulative influence of both endpoint nodes.

In Fig. 5, the required time for information to completely diffuse over the entire network is shown. Styles of the lines in

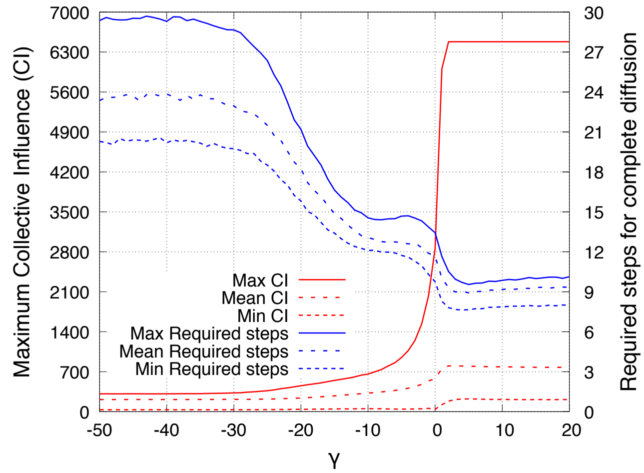
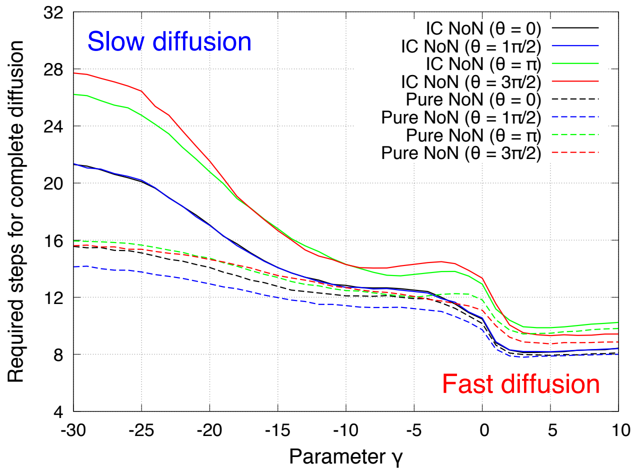


Figure 4. Collective influence and required steps for diffusion

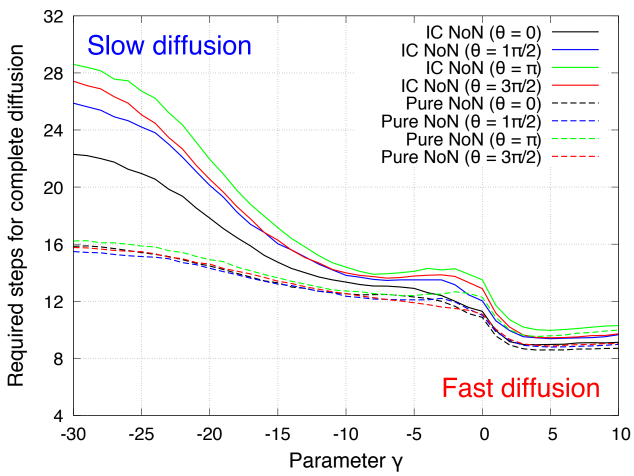
Fig. 5 basically correspond to the blue lines in Fig. 4 for the case of information diffusion in a subnetwork. As γ increases, influential nodes gradually appear and they minimize the diameter of each subnetwork in the interconnected network. However, the behavior of lines differ among each other, depending on the types of NoN models and the parameter θ. Regarding the parameter θ, we pick results of representative values in the range of $\theta \in [0, 2\pi)$, which sufficiently describes the characteristics of connectivity of interconnected networks.

The first striking point is that the diffusion speed greatly slows down when γ is small. This is because each subnetwork becomes uniformly connected as we confirme in Sec. IV-A. In such stretched subnetworks, the endpoints of interconnecting links in each network are located far away. The activation rule for outer-interfaces of the IC NoN model requires that both endpoint nodes of an interconnecting link have input when the outer-interfaces need to be activated. Therefore, the outer-interfaces tend to be turned off in interconnected networks composed of such stretched subnetworks. From those reasons, the solid lines of IC NoN marked much higher values than the dotted lines of Pure NoN. IC NoN achieves almost the same speed of information diffusion as interconnected networks whose interconnecting links are all inoperative, which can be interpreted from comparing Figs. 4 and 5.

On the other hand, IC NoN can achieve almost the same speed of information diffusion as Pure NoN. This can be seen when $\gamma \geq 2$ and $\theta \approx 0$. In this range of parameters, the interconnecting link of the diffusion source is assumed to connect the highest centrality nodes in each subnetwork. At the same time, both subnetworks have quite small diameter centered around those high centrality nodes. Therefore, strong diffusion sources enable quick information diffusion for IC NoN. It is notable that with this parameter settings, IC NoN can diffuse information as fast as Pure NoN, where interconnecting links are always active.



(a) Diffusion from interconnecting links with high centrality endpoint nodes



(b) Diffusion from randomly selected interconnecting links

Figure 5. Required steps for complete diffusion with changes in connectivity

VI. Conclusion and Future Work

In this study, we proposed an NoN model called IC NoN inspired by the Brain NoN model, which reproduces the activation rule of neurons of different modules that share interconnecting control links. As a basic characteristic, IC NoN does not allow unaccepted information to pass through interconnecting links, and thus prevents interconnected networks from unintended behavior. However, it is conceivable that interconnecting links themselves become sources of malicious information diffusion. Otherwise, in case of emergency, we can also start a diffusion of important information from the interconnecting links. Therefore, we investigated the configuration of connectivity within and between subnetworks, so that IC NoN can change the speed of information diffusion.

In the evaluation part, we simulated information diffusion starting at interconnecting links, while changing connectivity within and between subnetworks. The results showed that IC NoN can efficiently diffuse information as fast as Pure NoN, which does not consider the prevention of information diffusion between modules and thus proposes maximum diffusion speed with a given topology. We also found that even if malicious information spreads out from interconnecting links, we can reduce the diffusion speed to become as slow as interconnected networks with only inoperative interconnecting links, and thus IC NoN realizes robustness. Therefore, we can conclude that IC NoN can achieve both efficiency and robustness on information diffusion in interconnected networks.

In the evaluation of our proposal, we focused on the information diffusion starting from interconnecting links and we did not discuss the diffusion starting from a node or a link within a subnetwork, or interconnected networks composed of three or more subnetworks. Therefore, our future work would be to modify IC NoN or to configure a more complicated settings so that we can evaluate other types of information communication in interconnected networks.

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