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Title

Fractal Virtual Network Method for Achieving Efficiency and Robustness

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Abstract

In recent years, the types of network services provided through the Internet are increasing day by day, especially Internet of Things (IoT) services have been developed. A large number of devices have connected to the Internet. Network virtualization has attracted attention as a technology to flexibly accommodate various network services including IoT services. In order to provide IoT services with sufficient communication quality and availability by network virtualization, it is important to design virtual networks with high communication efficiency and high robustness against network failures. However, it is difficult to design virtual networks for IoT environment since there are a great number of devices. To obtain a guideline for designing virtual networks with high communication efficiency and high robustness, our research group has focused on fractal property. Our research group has preliminarily proposed a construction model of virtual networks with the fractal property and have verified that the virtual network constructed by the model achieves robustness against node failures. However, the previous construction model does not consider the physical distance between node pairs, which may lead to low communication efficiency on accommodating actual services. In this thesis, we investigate a design guideline of fractal virtual networks with high communication efficiency and robustness considering physical distances for networks with various sizes. In the evaluation of communication efficiency and robustness of the virtual network designed by our construction method, we show that the communication efficiency improves by more than 30%.

Keywords

Internet of Things (IoT)

Virtual Network

Fractal Property

Fractal Dimension

Hierarchical Modular Network

Contents

List of Figures

List of Tables

1 Introduction

In recent years, the Internet has provided various network services as an IT infrastructure that supports our daily lives. In network services, there are video streaming services, SNSs, cloud services and IoT services. Especially in IoT environment, the network scale is huge and different for each service, and then it is difficult for network administrators to control the whole of networks. Therefore, considering performance and management aspects of network services, demands of network administrators such as separating the traffic for each service and efficiently using the network equipment are increasing. Network virtualization technology has gained attention as one of technologies to meet these requirements. The network virtualization technology is a technique of logically dividing a physical network resource and constructing multiple virtual networks on a single physical network. By the network virtualization technology, physical infrastructure providers (InPs) provide an independent virtual network with different functions and performances to each service provider (SP), whereby the service providers can independently develop their own network services using their own virtual networks [1].

Virtual networks need high efficiency and robustness to provide network services with sufficient communication quality and availability because in the case of low efficiency, users of a network service can not enjoy the service with satisfactory communication quality and in the case of low robustness, virtual node/link failures do not allow node pairs to communicate with each other and unfortunately the service may result in failure. Hence, it is desirable that the virtual network holds high efficiency and high robustness. However, at present, there is no design guidelines for virtual networks with high efficiency and high robustness, and it is not clear what kind of virtual network SPs should build.

Then, we focus on fractal property as an approach for constructing large-scale virtual networks with high efficiency and high robustness. The fractal property is the property that the same structure and properties are seen even if the scale for observing the object is changed. In network science, when we assume that a certain node is a starting point and the hop length from that node increases, the increase rate of the number of reachable nodes from that node becomes constant. One of the criteria for quantitatively determining that a network has fractal property is that fractal dimension d_f has a finite value. By hop length l and the number of nodes M reachable from within l hops from a certain origin node, d_f is defined as

$$M \sim l^{d_f}. \tag{1}$$

Our research group has clarified that a topology with fractal property has a large number of redundant paths compared to a topology without fractal property and is robust against node failures [2]. Furthermore, we have devised a design model of a network topology with fractal property and confirmed that the fractal network using the design model shows excellent performance of failure tolerance [3]. However, most of the existing studies on fractal networks including [3] does not take physical aspects of networks into account when they devise network design model. Therefore, there is a problem that it could be impossible to accommodate the virtual network configuration based on the existing model on the physical network. For example, when you attempt to arrange a lot of virtual nodes in one physical node, it is conceivable that the upper limit of the number of virtual nodes that the physical node can accommodate is exceeded. Besides, when many virtual nodes are arranged between only nodes in distance on the physical network and the virtual links are constructed, that leads not only to waste the usable bandwidth of the physical networks but to make efficiency low even in communication of nearby nodes. Daqing et al. [4] suggest that the physical distance between nodes has strong relationship with d_f, and as the physical distance between the nodes becomes shorter, d_f becomes smaller. That is, we can express the physical distance by d_f. Actually, since the physical distance is not taken into account in the network using the model of the existing studies, its d_f takes a fixed value. Daqing et al. also suggest that d_f is related to the physical diffusion phenomenon, and as d_f increases, the information spreading speed in the information network, that is, the efficiency increases. For this reason, in the network using the existing design model, the information spreading speed is constant, and the average hop length and the diameter of the network increase as the scale increases, resulting in low efficiency. In order to solve the problem, in this paper, we devise a virtual network design method with d_f as the control parameter of cost, give the virtual network high efficiency and high robustness by considering the physical distance and the properties are kept high in IoT environment of different scales.

2 Related Work

2.1 Internet of Things

Internet of Things (IoT) is the future Internet where not only people but things can connect each other through the Internet [5]. IoT mainly consists of sensors, clouds, analytic systems and actuators. First, the sensors bring various information from circumstances around our lives such as temperature, sound or image. Second, the information is sent to the clouds through the Internet and is stored as big data in the clouds. Third, analytic systems (often artificial intelligences) analyze the big data, and they exploit and organize meaningful data from a meaningless series of numbers and letters. Fourth, according to analysis results, actuators begin to work, and they feed back various information to us or do some work automatically. In this way, IoT can improve our quality of lives.

In recent years, many SPs have developed own IoT services. Morandi et al. [6] focus on an urban IoT environment and design it to follow the Smart City vision. In the paper, they provide a comprehensive survey of the enabling technologies, architectures and protocols to realize the Smart City. Moreover, they mention the technical solutions for the Smart City and present best-practice guidelines for the design of it. Piyare et al. [7] proposes an extensible and flexible architecture for integrating wireless sensor networks with the cloud. As the testbed, they develop a REST-based Web service and realize remote monitoring system. It enables data access from anywhere and has the alert function that notifies users via email or tweets for monitoring data numbers when they exceed a certain threshold. Gachet et al. [8] launches the project for the health care of the elder people, using technologies associated with IoT. The main concern of the elderly is their health and its consequences depending on their self-rated ill health. Since the elderly have different health problems, doctors need to accurately diagnose each patients. However, the diagnostics is often dependent on how well the patients understand their ill health and that may lead incorrect results. Then, to correctly know the patients' body states, Gachet et al. use the sensors to quantitively calculate their states and develop the system that feeds back the state information to doctors.

Here, note that the scale of each network service is different and especially in IoT environment, the scale is often huge. The network of wireless sensor networks [7] is actually composed of thousands of sensors while the one of the Smart City [6] consists of millions or more of devices.

Therefore, there is a problem that it is difficult for network administrators to control the whole of a large and different scale of IoT networks and that causes inefficient network configuration.

2.2 Network Virtualization

Network virtualization is a technique that enables us to manage networks easily by dividing physical networks logically for each SP [9]. In detail, in the network virtualization, multiple virtual machines (virtual nodes) are activated on a single general-purpose server, the virtual nodes are connected by virtual links using a part of the bandwidth of the physical line and a virtual network is constructed. Virtual networks can be established on a single physical network unless the capacity of the physical machine is insufficient and it is guaranteed by security protocols that the virtual networks do not interfere with each other. Multiple virtual networks, therefore, can independently co-exist on the single physical network and network managers can operate a lot of different virtual networks for each SPs by system softwares for the operation such as OpenFlow [10].

Note that when SPs construct virtual networks, they need to consider the topologies of the networks and what structures the topologies hold such as the scale-free [11], small-world [12], hierarchical or modular structures [13]. The worst case is that they selfishly construct a virtual network ignoring the structure of it and that causes such low efficiency or low robustness that network services cannot work well. Therefore, it is important that in order to provide a sufficient quality of network services to users, network designers understand the structures of the network topologies and what effects given the network by the structures.

2.3 Virtual Networks for IoT Services

In this section, we introduce how virtual networks under IoT environment are designed in terms of topologies. As noted above, since the scale of the IoT networks is huge, it is impossible for humans to design the whole virtual network considering whether each link between all node pairs are established or not. In addition, there are few guidelines for designing such a large-scale network. Thus, network designers currently construct the large-scale network in their own way and they often do not comprehend what properties their networks have. In this paper, we present a new design model for such a large-scale network and show the guidelines to the network designers.

As an approach for designing the large-scale virtual networks, Yoshinobu et al. [3] focus on a fractal property. The fractal property is also called a self-similar property and means a property that a similar structure can be seen on any scale. In Yoshinobu's design method, they prepare an arbitrary network in advance and duplicate the same network as the first network. Then, the two networks are connected at some level of density, they further duplicate the same network as the newly constructed network and connect these two networks. After that, the above procedure is repeated. It is clear that the network designed in that way has the fractal property. However, the model excludes the physical aspects of the virtual network, that is, the model does not take into account the arrangement of virtual nodes in the physical network. As a result, when the virtual network is embedded into the physical network, it is possible that the virtual network is composed of a lot of long links from a viewpoint of the physical distance depending on the arrangement of virtual nodes and virtual links and the efficiency of the virtual network becomes low. The lack of physical concepts of networks is true not only for the method in [3] but for many of the existing studies, where the network topologies are built often based on only degree properties [11, 12, 14, 15]. Thus, there are few guidelines for designing the large-scale virtual networks currently and incorporating physical perspectives is a key point.

3 Method for Constructing Virtual Network with Fractal Property

In the existing model, since the physical distance is not taken into consideration, the fractal dimension, which is pointed out to be related to the communication efficiency in [4], is fixed, and the communication efficiency decreases as the network scale increases. Therefore, in a large-scale environment such as IoT, the efficiency of the network using the existing model is low. This chapter shows a generation algorithm that obtains a network whose fractal dimension is arbitrary, thereby confirming that arbitrary fractal dimensions can be actually obtained.

3.1 Relation between Fractal Dimension and Physical Distance

As noted already, similar structures can be seen on different scales in the network showing the fractal property. However, even among the fractal networks, there are different structures in terms of diffusion [16]. For example, if many nodes are allowed to connect only adjacent nodes, the diffusion speed is slow, while if even a few nodes can connect nodes at a distance, the diffusion speed rapidly gets faster. That sounds the small-world property but the difference between the small-world and the fractal is that there is no bias in the fractal network from a viewpoint of the degree. The small-world network often has the scale-free property and so-called 'hub nodes' shrink the diameter of the network. However, if the 'hub nodes' are broken, the damage is very serious and it is possible that the entire network collapses. On the other hand, since the fractal network has no bias in the degree distribution, the damage to a part of the network anywhere seldom causes serious situations, as the objects with the fractal property in the natural world is robust [17]. That is, the fractal network has different efficiency by controlling link length distribution and is robust against node failures at some level. The link length distribution of the fractal network, related to the diffusion, is characterized by a fractal dimension.

As a method for quantitatively evaluating the fractal dimension, Box-Counting Algorithm or Cluster-Growing Algorithm is widely used. In the Box-Counting Algorithm, nodes that can reach each other within a certain hop length are called Boxes, and the number of Boxes for covering the entire network with non-overlapping Boxes is counted. However, since there are countless Box patterns that can cover the entire network, various heuristic methods have been proposed [18], but since it is impossible to obtain a stable solution in a large-scale network, it is unsuitable for strict fractal analysis. On the other hand, in the Cluster-Growing Algorithm, the number of nodes reachable from an origin node within a certain hop length is counted. Although different results are obtained depending on the selection of the origin node, a method of obtaining a stable solution by taking statistics of results at a lot of origin nodes is generally used in the complex network field. Since it is mathematically proved that the result by Box-Counting Algorithm and the result by Cluster-Growing Algorithm are equivalent, we assume fractal analysis using Cluster-Growing Algorithm in the following.

Equation (1) is commonly used to obtain the fractal dimension, but since the focus is on the hop length in the construction model of Chapter 3.2, It is necessary to redefine it as the link length r instead of the hop length l as follows:

$$M \sim r^{d_f}. \tag{2}$$

Using Equation (2), we obtain d_f by following the procedure below.

1. Select a node to be an origin (hereinafter, the origin node).
2. Let the set of nodes adjacent to the origin node be shell 1, the node set adjacent to shell 1 and not included in the previous shell be shell 2, and repeat the procedure until it covers all nodes.
3. Measure the following two.
   (a) The average Euclidean distance r between the nodes in shell s and the origin node.
   (b) The number of nodes $M(s)$ in shell s.

We execute the above procedure on multiple origin nodes and average the results. By definition, s is the shortest path length between the origin node and any nodes in shell s. An example of applying this procedure to both a square lattice network and a tree network is shown in Figure 1. The fractal dimension is determined from Equation (2).

3.2 Procedure for Fractal Virtual Network Construction

Daqing et al. [4] suggest that when the link length distribution, viewing the network in physical space, follows a power distribution, the fractal dimension is found in the scaling exponent of that distribution. Therefore, we aim to generate a network showing the desired fractal dimension by giving the scaling exponent as a parameter. A specific generation algorithm is shown below.

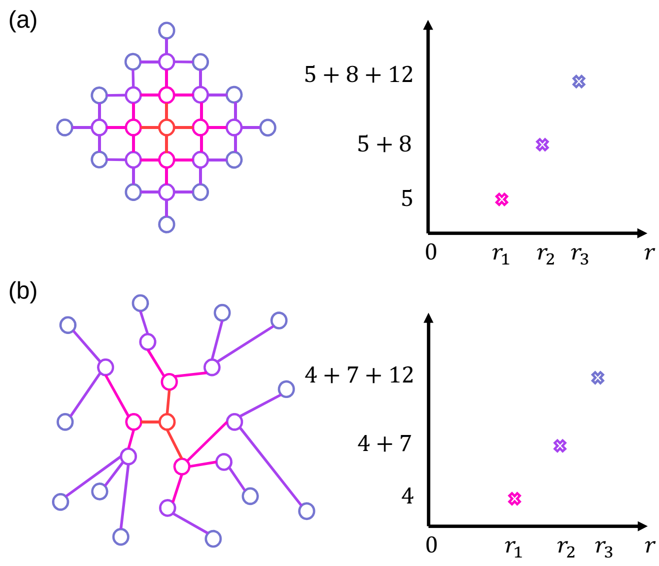


Figure 1: Calculation examples of r and M for a square lattice network (a) and for a tree network (b) [4]

Parameters to be input to the algorithm The number of nodes n, the average degree λ of the network to generate, the exponent δ of link length following power distribution.

Step 1. Arrangement of nodes Arrange n nodes in a two-dimensional square lattice, but do not construct any links here. Let the distance between the nearest nodes be 1. Let the node ID of each node be $0, 1, 2, ..., n - 1$.

Step 2. Determination of the degree of each node Select node i ($\in \{0, 1, 2, ..., n - 1\}$) randomly and repeat from Step 2.1 to Step 2.2. for all nodes below.

Step 2.1. Generate an integer random number p_i according to Poisson distribution $P(\lambda)$.

Step 2.2. Set p_i as the degree of node i.

Step 3. Establishment of links Select node i ($\in \{0, 1, 2, ..., n - 1\}$) randomly and check the number of nodes connected to i. If the number of nodes already connected to i is less than p_i, continue from Step 3.1. to Step 3.5. until the number of nodes connected to i is equal to p_i. Perform the above procedure for all nodes.

Step 3.1. Generate a random number q based on the probability distribution function $Q(r)$ following a power distribution

$$Q(r) = cr^{d-1}r^{-\delta}, \tag{3}$$

where $d = 2$ since d is the dimension of Euclidean space that embeds the network into, c is a normalization factor to adjust to satisfy $\int_1^L Q(r)dr = 1$ and $L = \sqrt{n}$ because L is the upper limit of the power distribution.

Step 3.2. Let q be the link length.

Step 3.3. Create the node group of the connection destination candidate of node i. Calculate the node group excluding the node i itself from all the nodes and the node already connected to the node i as the node group of the connection destination candidate.

Step 3.4. Select the node closest to the link length q from the node i from the node group of the connection destination candidate. If there are multiple nodes at the same distance, select one at random. Let u be the selected node.

Step 3.5. Check the degree of u and the number of nodes already connected to u. If the number of nodes already connected to u is less than p_u, build a link between node i and u. Otherwise, remove u from the connection destination candidate and return to Step 3.4.

3.3 Topological Properties of Fractal Virtual Networks

Degree Distribution Analysis

Since degrees of all nodes are referred to in Step 3 of the algorithm of Chapter 3.2, it is guaranteed that the degree distribution given to our model is the same as the degree distribution of the generated network.

Link Length Distribution Analysis

In the network construction algorithm of Chapter 3.2, the link length distribution (LD) of the network is strictly different from the LD given as an input to our model. The reason is why if it is repeated multiple times that the node closest to the link length q from the node i is removed from the connection candidates in steps 3.4 to 3.5 in the algorithm of Chapter 3.2, a link having a link length different from the original link length q is constructed. If the LD given to the generation algorithm and the LD of the generated network are significantly different, it is impossible to obtain a network showing the fractal dimension to be expected by originally changing the scaling exponent δ of the power distribution given as the LD. In addition, It is impossible to evaluate the relationship between the fractal dimension and properties such as efficiency and robustness. Therefore, the difference between the LD given to our model and the LD of the generated network is evaluated and we verify that the difference is acceptable.

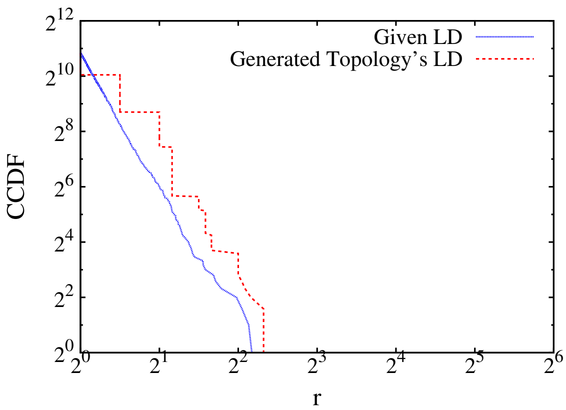
As the inputs of our model, we give the number of nodes $n = 900$ and the average degree $\lambda = 4$, the exponent of the required LD $\delta = 1.0, 2.0, 3.0, 4.0, 5.0$. The evaluation result is shown in Figure 2. The horizontal axis represents the distance r, and the vertical axis represents the complementary cumulative distribution function (CCDF) with the logarithmic scale of base 2. The LD given as the input of our model is expressed by the blue line and the red line indicates the LD of the generated network. Looking at Figure 2, there is no big difference in inclination of both lines as a whole. Therefore, we believe that it is possible to control the fractal dimension by giving various values of δ to our model. However, as the value of δ gets larger, there is a larger difference between the values indicated by both lines. The reason for this is why if we give our model a large value of δ, our model tries to construct more short-distance links, but there is a limit to the number of short-distance links that can be constructed. Therefore, even if a value greater than $\delta = 5$ is given to our model, it is considered that the LD of the generated network hardly changes.

The conclusion of this section is that although it is somewhat different depending on the value of δ, it is verified that it is possible to change the slope of the generated network by changing δ. Therefore, it is possible to control the fractal dimension using our model.

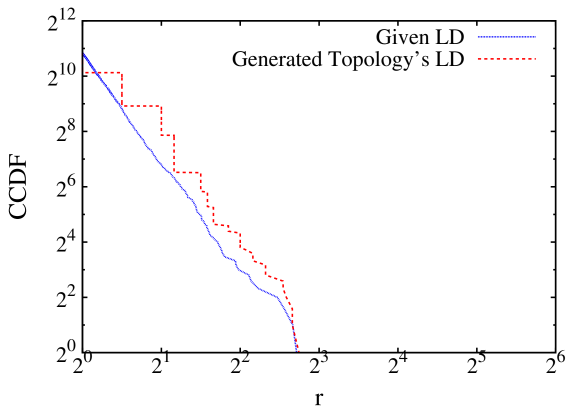
Fractal Dimension

We analyze whether the network using our model has fractal properties, and if so, how the fractal dimension d_f correlates with the parameter δ. We prepare networks by giving the values of $n = 1024$, $\lambda = 4$, $\delta = 1.1, 2.0, 2.5, 2.6, 3.0, 3.5, 4.5$ to our model. For all the network we parepare, the fractal dimensions are calculated by the method of Chapter 3.1. By checking whether each fractal dimension is different or not, we verify that a network showing arbitrary fractal dimension can be obtained by changing δ.

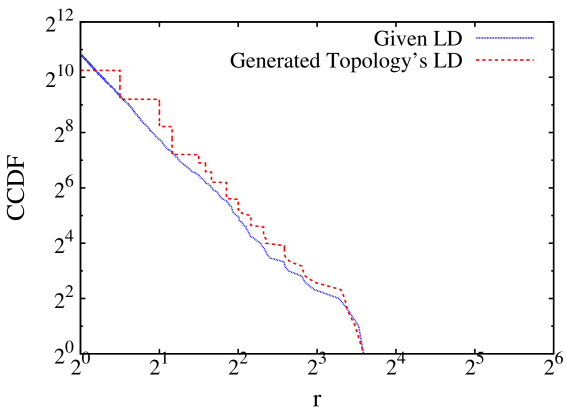
The result of the analysis is shown in Figure 3. The horizontal axis of Figure 3 is the distance r, and the vertical axis is the number of reachable nodes M with r. By setting both axes to a logarithmic scale, the slope of the straight line composed of the obtained data points shows the fractal dimension. As a result, the fractal dimension of the network of $\delta = 1.1$ is about 4.0, the fractal dimension of the network of $\delta = 4.5$ is about 2.0, and the fractal dimensions of the other networks increases as δ decreases. The relationship between δ and the slope is clear, which shows that it is possible to evaluate the relationship between the fractal dimension and efficiency or robustness in the next chapter.



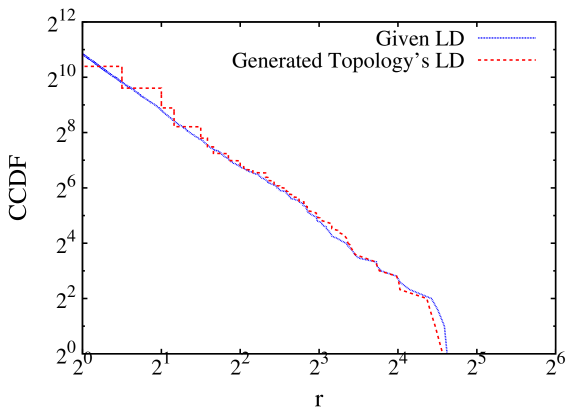
(a) $\delta = 5.0$



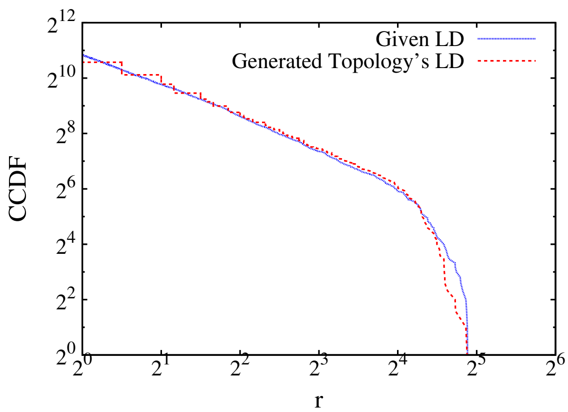
(b) $\delta = 4.0$



(c) $\delta = 3.0$



(d) $\delta = 2.0$



(e) $\delta = 1.0$

Figure 2: Link length distribution

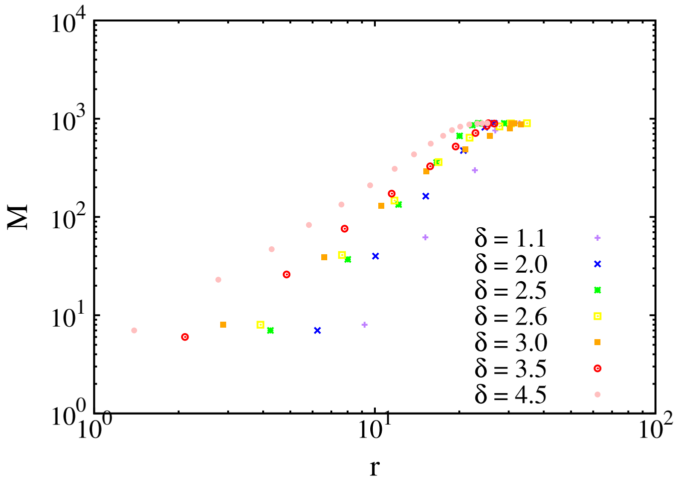


Figure 3: The fractal dimension of networks generated by giving each δ to the algorithm

4 Efficiency and Robustness of Fractal Virtual Networks

Through the evaluation in this chapter, we verify that our model is effective as a design guideline for a large-scale network because the virtual network based on our model shows high efficiency (efficiency) and high robustness and shows high performance on various scales .

4.1 Metrics for Efficiency and Robustness

In the evaluation, the efficiency and robustness of each network is evaluated. To do this evaluation, we refer to the information centrality C_S^I [19] expressed as

$$C_S^I = \frac{\Delta E}{E} = \frac{E(G) - E(G')}{E(G)}, \tag{4}$$

where C^I is information centricity, S is a set of failure nodes, G is a pre-failure graph, G' is a post-failure graph. $E(G)$ is defined as

$$E(G) = \frac{1}{n(n-1)} \sum_{i \neq j \in G} \frac{1}{d_{ij}} \tag{5}$$

where n is the number of nodes in the graph G and d_{ij} is the shortest path length between i and j. The information centrality is an metric showing how much the node group included in a certain range contributes to the entire network from a viewpoint of efficiency. If a network collapse due to node failures, the shortest path length of two nodes between the divided subnetworks is assumed to be ∞. In addition, since the deletion of a node leads to increasing the shortest path length between multiple node pairs, $E(G') \leq E(G)$ always holds.

However, the information centricity C_S^I is an metric indicating how much S contributes to the entire network from the viewpoint of efficiency by removing S from the network. That is, C_S^I indicates the importance of the node group S, which is different from the meaning of robustness. In this case, it is inappropriate to use C_S^I that focuses on the failed node group in order to evaluate how much the remaining node group can maintain the communication function when the node group fails. Attention to the failed node group is expressed as $\sum_{i \neq j \in G} \frac{1}{d_{ij}}$ of Equation (5) and the efficiency before node failures contains the reciprocal of the shortest path length from/to the failed node group. Therefore, the robustness metric R_{S_r} shown in Equation (6) and the partial Efficiency E_{S_r} shown in Equation (7) are obtained by correcting the equations not to contain the reciprocal of the shortest path length from/to the failed node group as following

$$R_{S_r} = \frac{\Delta E_{S_r}}{E_{S_r}} = \frac{E_{S_r}(G) - E_{S_r}(G - S_r)}{E_{S_r}(G)} \tag{6}$$

and

$$E_{S_r}(G) = \frac{1}{n(n-1)} \sum_{i \neq j \in G - S_r} \frac{1}{d_{ij}(G)} \tag{7}$$

where S_r is a node group included in the failure radius r. This makes it possible to calculate the change of efficiency only in the nodes other than S_r.

4.2 Evaluations

In this chapter, we evaluate how efficient and robust the fractal virtual network is under node failures.

4.2.1 Comparison with Random Network under Unified Cost Conditions

In this section, Efficiency and R_{S_r} are evaluated using a network that unifies link construction costs. We verify that fractal networks is superior to non-fractal ones in terms of efficiency and robustness. Here, the cost means the sum of the physical link lengths and the number of links. The network to compare and evaluate is a network of $\delta = 1.1$ (hereinafter Fractal$_{\delta=1.1}$) and a random network (hereinafter Random). The reason for using Random is to consider the advantages of the link length distribution being a power distribution from the viewpoints of efficiency and robustness because the difference between Fractal$_{\delta=1.1}$ and Random is the power distribution or the exponential distribution of the link length distribution.

Networks for Evaluation

The network to be evaluated is as follows.

- **Fractal$_{\delta=1.1}$:** Fractal$_{\delta=1.1}$ is a network generated by giving $n = 900$, $\lambda = 4$, $\delta = 1.1$ to the generation algorithm in Chapter 3.2.

- **Random:** Random is a random network based on ER model. As a method of constructing Random, first, n nodes are set in a two-dimensional square lattice and then randomly select two nodes in that network. Second, we calculate the Euclidean distance of the two nodes and decide whether to construct the link using the probability density function according to the exponential distribution of the average μ. After that, we randomly select two nodes and decide whether to build links until the number of links is equivalent to the number of links in Fractal$_{\delta=1.1}$. In order to unify the cost (the sum of the physical distances), μ is set to the average value of the physical distance of Fractal$_{\delta=1.1}$, here μ is 8. Also, we construct links avoiding self loops and overlapping links.

Failure Scenario

When a node is selected as a center of node failures, the nodes within the Euclidean radius r from the selected node and the associated links to the generation network are excluded (hereinafter range failure). In this evaluation, all nodes are selected as the center of failed nodes O and range failures occur for $r = 0, 2, 4$ from each node.

Result

The evaluation result of efficiency is shown in Figure 4. Labels of each network are appended to the horizontal axis and the efficiency of Equation (5) is shown on the vertical axis. As a result, we found that the network efficiency of Fractal$_{\delta=1.1}$ is about 0.05 higher than Random. Considering this difference from the viewpoint of the hop length, in Fractal$_{\delta=1.1}$, all node pairs can be reached to each other with the efficiency of 0.22, that is, with an average hop length of about 5 hops, while Random has the efficiency of 0.15, that is, an average of about 6.5 hops. The difference is about 1.5 hops.

Next, the evaluation result of R_{S_r} is shown in Figure 5. The horizontal axis shows the label of each network, and the vertical axis shows the distribution of R_{S_r} with a box-and-whisker. r is the failure radius, and the box-whisker chart shows the distribution of R_{S_r} when range failures happen in the network with the failure radius of each of (a), (b) and (c) for the center node of failure 900 nodes (all nodes). The points in each graph indicate average values. As a result, we find that R_{S_r} of Fractal$_{\delta=1.1}$ is smaller when R_{S_r} is viewed as the maximum value or the average value, that is, Fractal$_{\delta=1.1}$ is more robust. The reason why the R_{S_r} of Fractal$_{\delta=1.1}$ becomes smaller is because the stable reachability of the shortest path between node pairs at short distance is greatly different. Here, the stable reachability of the shortest path indicates such property that when nodes fail, the shortest path of node pairs not including the failure nodes does not switch. The link length distribution of Fractal$_{\delta=1.1}$ follows the power distribution of the scaling exponent -1.1 while the link length distribution of Random follows the exponential distribution of the average 8. Thus, there are many links around the link length 8 in Random, but in Fractal$_{\delta=1.1}$ the number of links decreases as the link length becomes longer from 1. Therefore, in Fractal$_{\delta=1.1}$, there are many short-distance links physically and reachable to short-distance nodes with short-distance links, but in Random, in order to reach short-distance nodes, medium distance is needed by passing through multiple nodes and links. As a result, the shortest path that physically arrives at a short distance node when a failure occurs is almost unchanged at Fractal$_{\delta=1.1}$, but it can be considered that the shortest path varies greatly in Random.

4.2.2 Efficiency and Robustness of Networks with Different Fractal Dimensions

In this section, we evaluate the efficiency and robustness of networks with various fractal dimensions. This clarifies the relationship between cost, efficiency and robustness when the fractal dimension is regarded as cost parameter for the network design.

Networks for Evaluation

We use networks generated by giving $n = 1024$, $\lambda = 4$, $\delta = 1.1, 2.0, 2.5, 2.6, 3.0, 3.5, 4.5$ to the generation algorithm of Chapter 3.2. Besides, we use a grid network Grid that accounts for $\delta = \infty$.

Failure Scenario

We assume range failures for a center of all nodes as noted already. Also, range failures occur for $r = 0, 2, 4$ from each node.

Result

The evaluation result of Efficiency is shown in Figure 6. The horizontal axis represents the exponent δ of the link length distribution to be given our model and the vertical axis represents the efficiency of Equation (5). Looking at figure 6, in the range of δ in $[1, 3]$, there is a negative linear relationship between δ and efficiency. However, in the range of δ in $[3, 4.5]$, the slope becomes gentle. The reason is why as the value of δ increases, the network approaches the grid shape and the sum of link lengths gradually approaches the minimum value at some δ value. In addition, compared with the fractal network of $\delta = 4.5$ and Grid which is a two-dimensional lattice network, it is seen that the efficiency of the fractal network is more than twice as it can be achieved

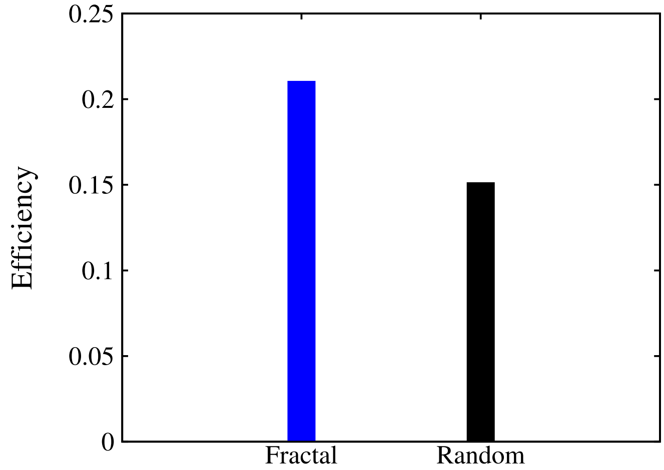


Figure 4: Comparison of efficiency with both Fractal and Random networks

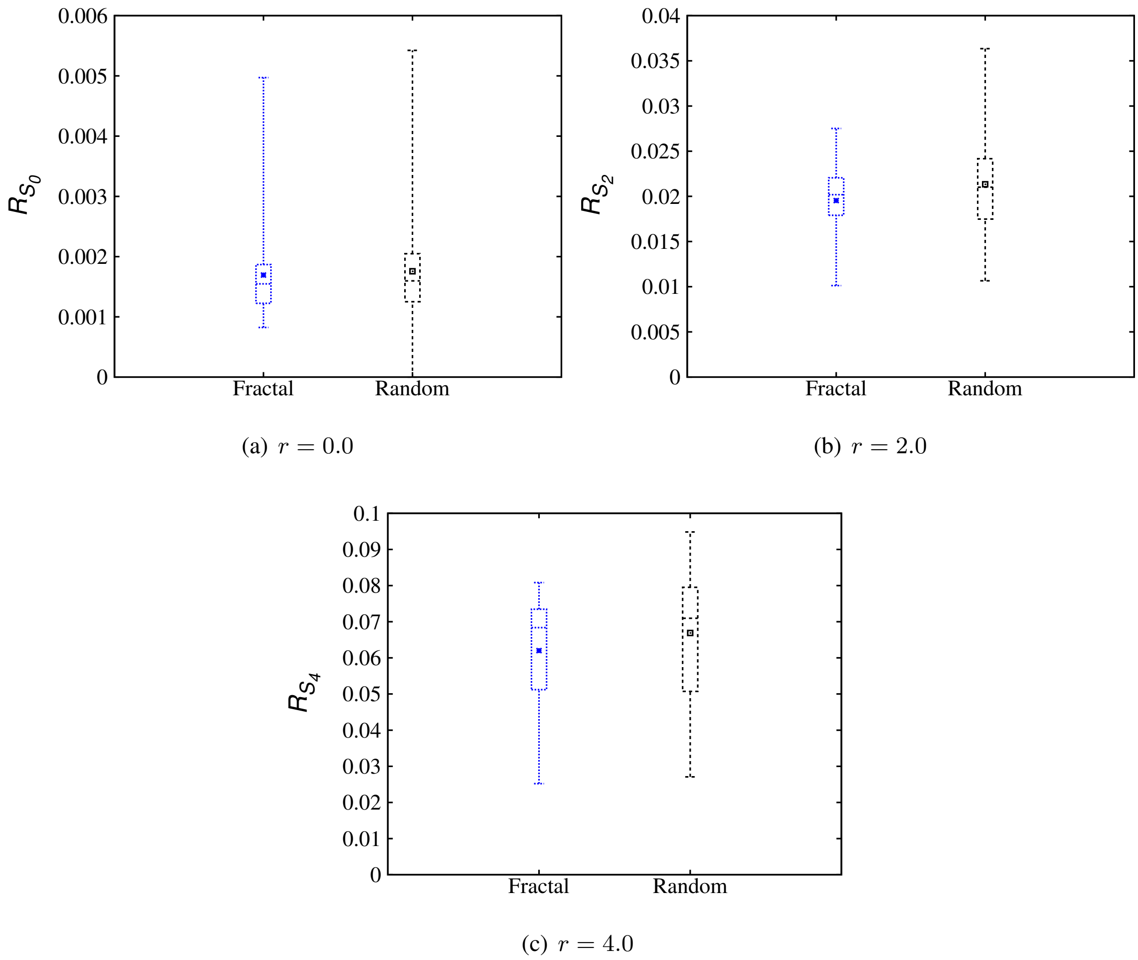


Figure 5: Comparison of R_{S_r} with both Fractal and Random networks

by only including a small number of long distance links. Since the shape of Grid's network is close to that when δ is increased, the results of Grid's efficiency are displayed where the value of δ is large. In conclusion, it turned out that the efficiency increases as δ decreases (the fractal dimension increases).

The evaluation result of the relationship between δ and R_{S_r} in each r is shown in Figure 7. In the evaluation, we calculate the robustness metric R_{S_r} when range failures occur for all nodes as the center of failure O. Since R_{S_r} represents the change in efficiency other than the failure node group S_r, it means that the smaller the value of R_{S_r}, the more robust it is. The horizontal axis shows the distribution of the exponent δ of the required link length distribution of the generation network and the vertical axis shows the distribution of R_{S_r}. The points in each δ mean the average value. Looking at figure 7, the maximum value of R_{S_r} increases with the increase of δ. For this reason, it can be seen that for networks close to the grid shape having a large value of δ, the network is seriously damaged depending on the failure location compared with the failure scale. The reason for this is that in a network where δ is large, there are few long-distance links in the entire network, and if the node holding the long-distance link fails, the efficiency is greatly reduced. Also, the result of Grid supports this idea. In other words, Grid without long distance links has low efficiency and there is no room for large change in R_{S_r}. Next, when looking at Figure 7, R_{S_r} is not changed at all at any failure scale in (a), (b) and (c). We consider that this stability against δ is brought by the fractal property that the fractal structure originally have. That is, since the fractal structure has the same structure anywhere at any scale, the same robustness is seen even if any nodes fail.

As a consideration, we evaluate hierarchical module property of what connection structure the high efficiency and high robustness of the network generated by our method (hereinafter generated network) are brought about . As an evaluation means, the evaluation is performed referring to [13] and expressing a matrix relating to link density in a network by a temperature graph. We make a concrete explanation using Figure 8 and Figure 9. Figure 9 is the adjacency matrix corresponding to the network in Figure 8 and each element ρ_i $(i = 1, 2, 3)$ is the link density, which is the value obtained by normalizing the number of links in and between the modules with the number of all the node pairs in and between the modules. For example, when looking at the hierarchy of $l = 0$, each module ID corresponds to row and column, and the link density in all modules is ρ_1. Next, in the hierarchy of $l = 1$, the number of links of each module pair is 2, and the link density is

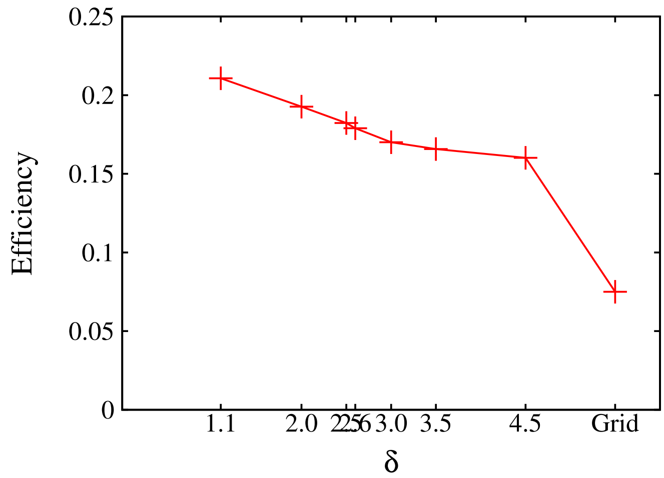
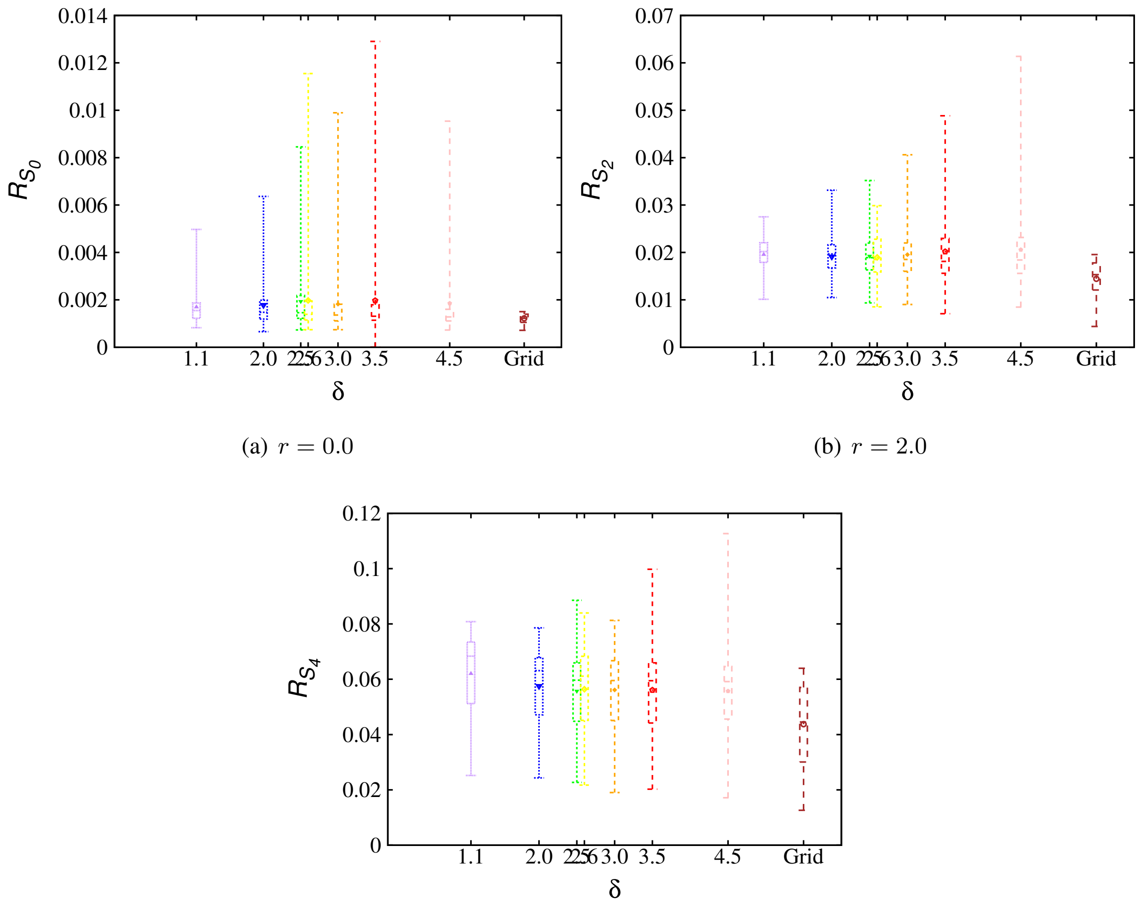


Figure 6: Efficiency of a generated network with each δ



(a) $r = 0.0$

(b) $r = 2.0$

(c) $r = 4.0$

Figure 7: Relationship between δ and R_{S_r}

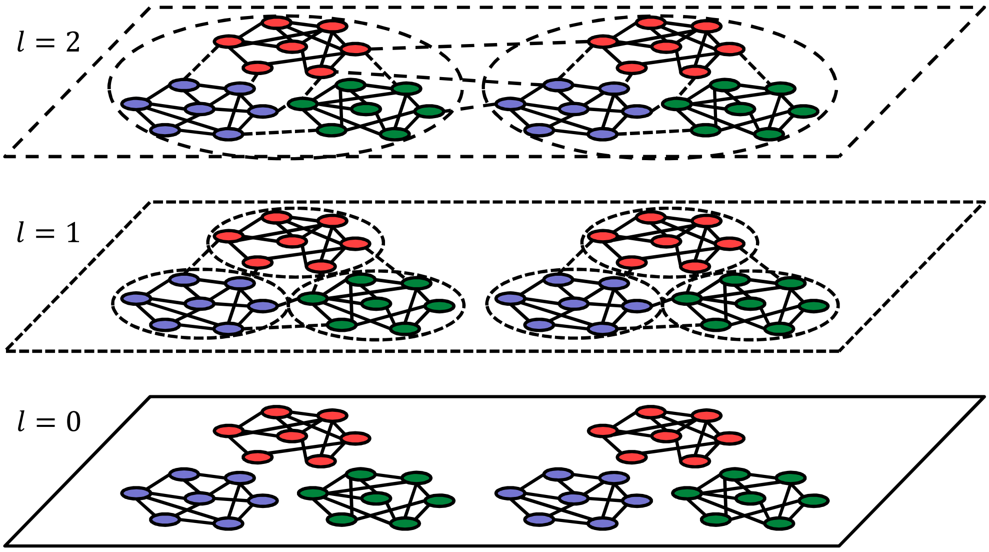


Figure 8: Evaluation method for hierarchical modularity: An example of hierarchical module structure

Table 1: Procedure to determine module ID

0	1
2	3

ρ_2. The condition for being a hierarchical module structure is to satisfy $\rho_1 \geq \rho_2 \geq \rho_3 \geq ....$ In this evaluation, it is evaluated whether or not the generated network satisfies the above condition, compared with other networks.

As a method of determining the Module ID corresponding to the row and column of the adjacency matrix, the entire network is divided into four such as Table 1, and Module ID are determined in order from the upper left. In addition, the Module ID at the hierarchical level below is divided into 4 sections of Table 1, which are 0, 1, 2 and 3 in the same way. Further, we divide one section into four, and make it 4, 5, 6, and 7. In this way, Module IDs are allocated while repeating recursively division, thereby the Module IDs are determined at each hierarchical level.

For hierarchical modularity evaluation, the following four networks are prepared.

- **Fractal$_{\delta=1.1}$:** Fractal$_{\delta=1.1}$ is a network generated by giving $n = 1024$, $\lambda = 4$, $\delta = 1.1$ to the generation algorithm in Chapter 3.2.

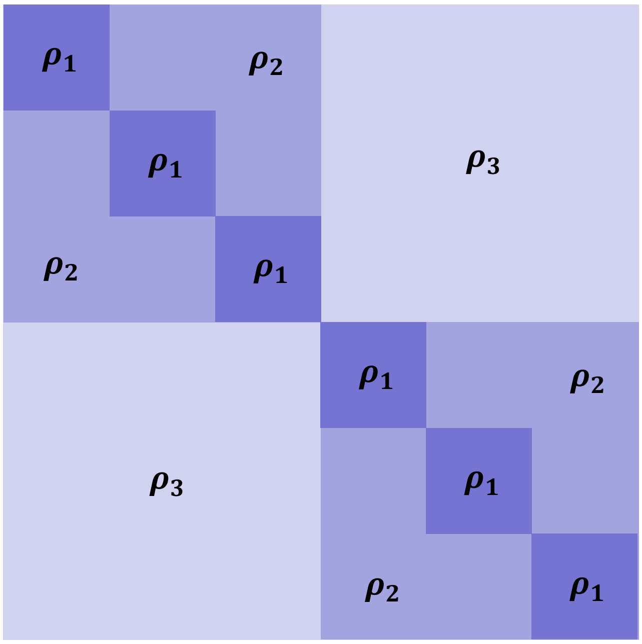
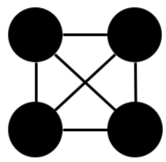
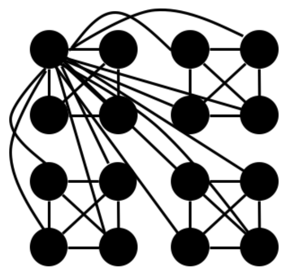


Figure 9: Evaluation method for hierarchical modularity: An example of the adjacency matrix corresponding to the hierarchical module structure in Figure 8

- **Fractal$_{\delta=4.5}$:** Fractal$_{\delta=4.5}$ is a network generated by giving $n = 1024$, $\lambda = 4$, $\delta = 4.5$ to the generation algorithm in Chapter 3.2.
- **Random:** Random is a random network based on ER model. As a method of constructing Random, first, n nodes are set in a two-dimensional square lattice and then randomly select two nodes in that network. Second, we calculate the Euclidean distance of the two nodes and decide whether to construct the link using the probability density function according to the exponential distribution of the average μ. After that, we randomly select two nodes and decide whether to build links until the number of links is equivalent to the number of links in Fractal$_{\delta=1.1}$. In order to unify the cost (the sum of the physical distances), μ is set to the average value of the physical distance of Fractal$_{\delta=1.1}$, here μ is 8. Also, we construct links avoiding self loops and overlapping links.
- **Pure:** Pure is a network that has both scale-free and hierarchical properties [20]. By using the network construction model of [20], it is possible to generate a network with a beautiful hierarchical structure. By comparing with Pure, we know the upper limit of hierarchy that



(a) $n = 0, N = 4$



(b) $n = 1, N = 16$

Figure 10: Procedure to construct hierarchical modular structure

Fractal can take. we explain the generation procedure of the hierarchical modular structure with a refinement so that it can be applied to a network in which nodes are arranged in a two-dimensional lattice using the Figure 10 . In Figure 10, n is the number of steps and N is the number of nodes. At the 0th step of (a), we connect four nodes so that the link density becomes 1. Next, in the first step of (b), three new networks of the previous step (a) are duplicated and arranged in a two-dimensional square lattice. Besides, we construct links from the other nodes to the node located at the top left so that they do not overlap. Thereafter, the network of the previous step is newly duplicated and arranged in a two-dimensional square lattice and the procedure of constructing a link from all the nodes to the node located at the top left position is repeated until the number of nodes in the network reached the number of given nodes.

The results of the evaluation are shown in Figure 11, 12, 13 and 14. First, comparing (a), (b), (c) and (d) of Figure 11 and 12, the proportion of yellow and black is more in Figure 11, ant the difference in value clearly appears. This is because Fractal$_{\delta=4.5}$ is composed of shorter link lengths and nodes at short distance are more densely connected while nodes at long distance are more sparsely connected. In particular, paying attention to (c), it turns out that Fractal$_{\delta=4.5}$ is more modular than Fractal$_{\delta=1.1}$, which indicates that the module structure can be seen at the lower hierarchical level. Next, looking at Figure 13, the value of link density between modules at long distance frequently exceeds the link density between modules at short distance, so it does not show hierarchical module property. In particular, as is clear from (d), no modular structure is taken at the low hierarchical level. In Random, a large number of links with an average link length of 8 are constructed, but on the other hand, the number of links with a link length of 1 is very small, and at a hierarchical level with a small module size such as (d), there frequently are no links. Finally, looking at Figure 14, Module 0 and all the other modules are always connected with a constant link density, such a structure is seen locally, and we can see that the modularity is high from the clear difference of colors. In other words, it turns out that the network takes the hierarchical module structure as expected. However, the hierarchical module structure regularly constructed in this manner has scale-free property, and although it is robust against random node failures, it is considered that it is weak against selective failures to hub nodes, which have a large number of degrees. That is, in the worst case of failures, it is possible that the network is seriously damaged. In conclusion, the fractal network takes the hierarchical modular structure and has different modular size per fractal dimension.

4.2.3 Efficiency and Robustness on Different Scale of Networks

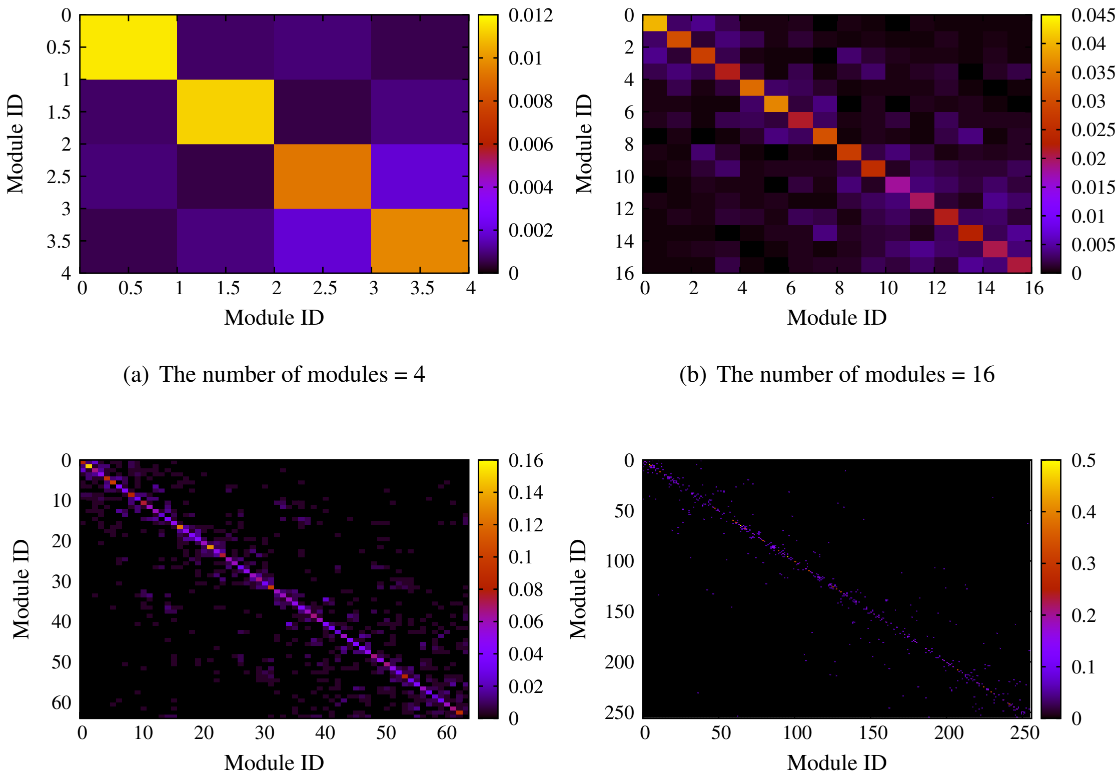
It is considered that the scale of the virtual network under the IoT environment is not only large, but the scale differs for each IoT service. In the previous evaluations, we showed that the virtual network with the fractal property has high efficiency and high robustness, but the network scale at that time was always constant. Therefore, under the IoT network environment where various scales are expected, the fractal network using our model is not always effective. Therefore, by evaluating efficiency and robustness of the fractal networks which give various parameters of the number of nodes to our model, we show the fractal network is effective at any scale in terms of efficiency and robustness.

Networks for Evaluation

We prepare the following networks for evaluation.

- **Fractal$_{\delta=1.1}$:** Fractal$_{\delta=1.1}$ is generated by giving $n = 256, 1024, 2304, 4096, 6400, 9216,$ $12544, 16384$, $\lambda = 4$ and $\delta = 1.1$ to the generation algorithm of Chapter 3.2.

- **Random:** Random is a random network based on ER model. As a method of constructing Random, first, n nodes are set in a two-dimensional square lattice and then randomly select two nodes in that network. Second, we calculate the Euclidean distance of the two nodes



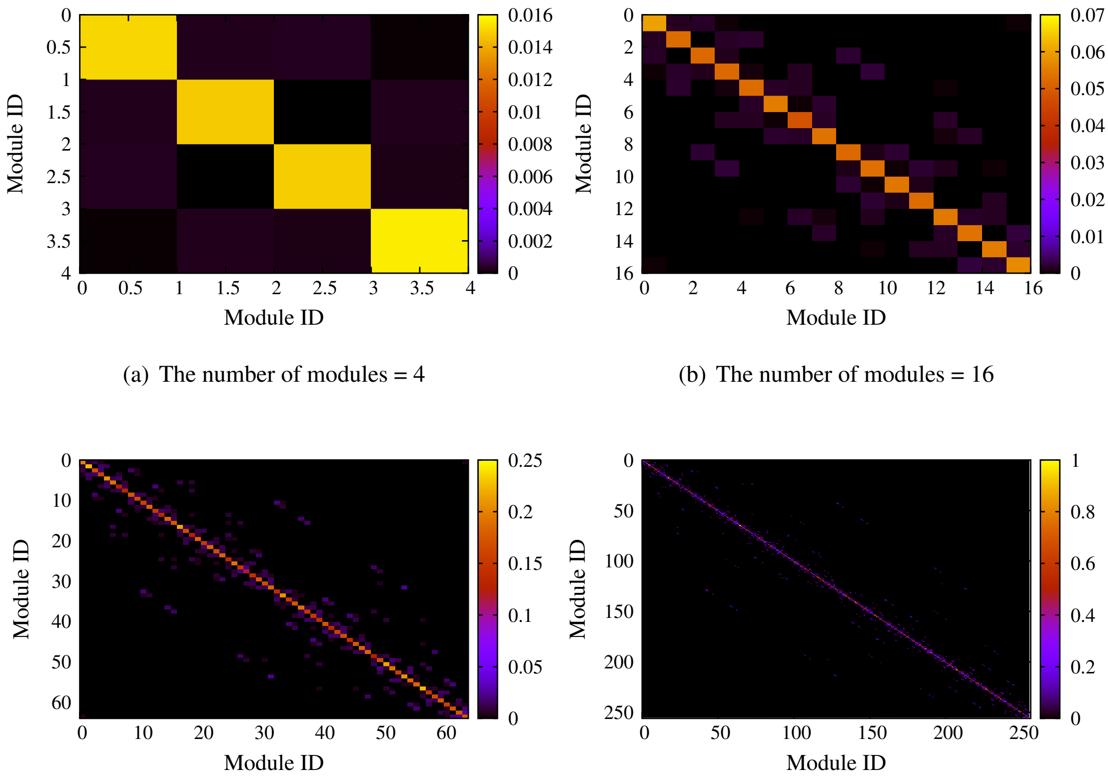
(a) The number of modules = 4

(b) The number of modules = 16

(c) The number of modules = 64

(d) The number of modules = 256

Figure 11: Hierarchical modularity of the fractal network with $\delta = 1.1$



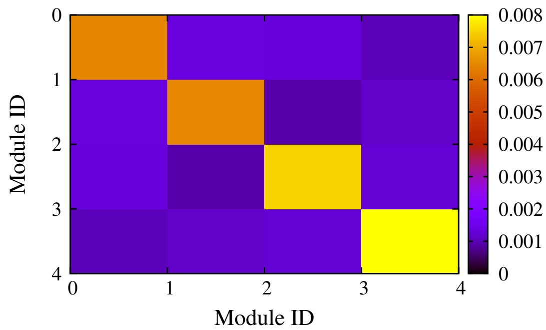
(a) The number of modules = 4

(b) The number of modules = 16

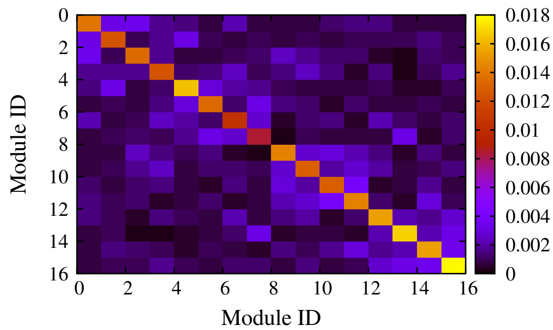
(c) The number of modules = 64

(d) The number of modules = 256

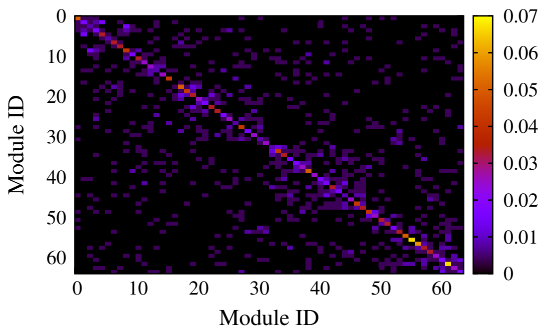
Figure 12: Hierarchical modularity of the fractal network with $\delta = 4.5$



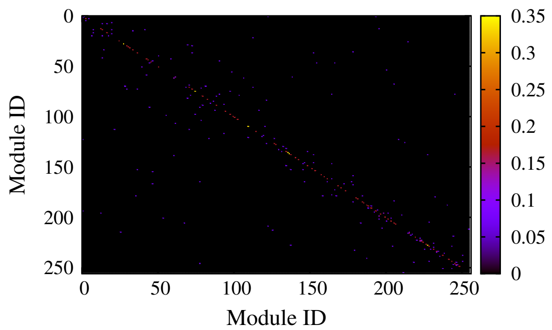
(a) The number of modules = 4



(b) The number of modules = 16

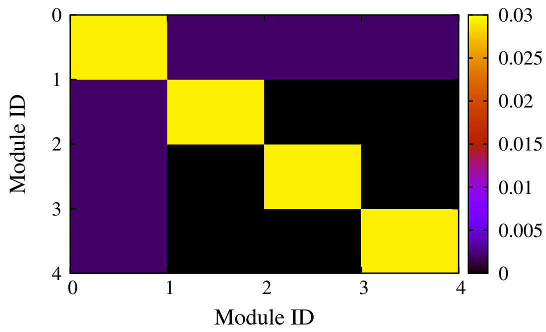


(c) The number of modules = 64

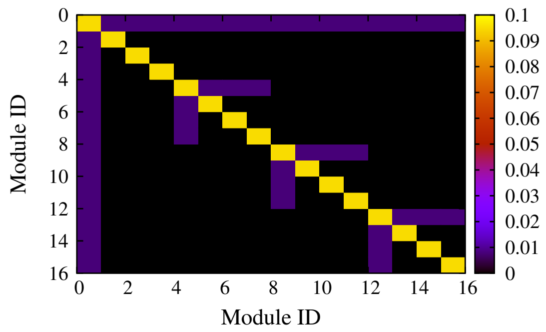


(d) The number of modules = 256

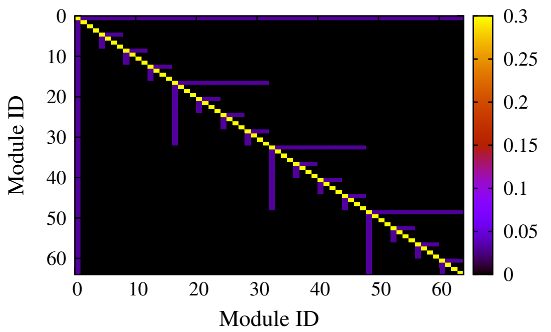
Figure 13: Hierarchical modularity of a random network



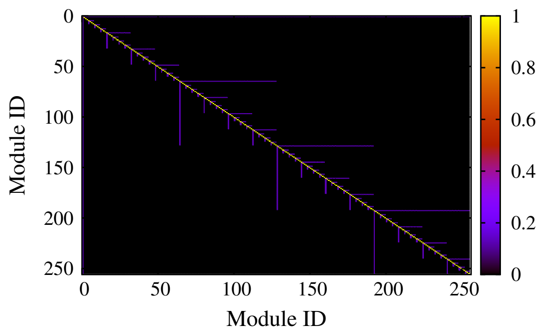
(a) The number of modules = 4



(b) The number of modules = 16



(c) The number of modules = 64



(d) The number of modules = 256

Figure 14: Hierarchical modularity of a hierarchical modular network

and decide whether to construct the link using the probability density function according to the exponential distribution of the average μ. After that, we randomly select two nodes and decide whether to build links until the number of links is equivalent to the number of links in Fractal$_{\delta=1.1}$. In order to unify the cost (the sum of the physical distances), μ is set to the average value of the physical distance of Fractal$_{\delta=1.1}$. Also, we construct links avoiding self loops and overlapping links.

Failure Scenario

The larger the scale of networks is, the larger range failures occur. In this evaluation, the scale of networks is different. That is why when we assume range failures of the same scale, influences of the failures decreases as the network scale is larger. Then, inevitably the larger network will be robust, but it is a mistaken interpretation. Therefore, by making the scale of the network proportional to the size of the range failures, robustness is evaluated so that the effects of range failures are equal among networks of different scales. The failure radius $r(n)$ of range failures is assumed to be $r(n) = \sqrt{n}/c$, using the number of nodes n and the constant c. In this evaluation, $c = 8$ is set. Also, the value of $R_{S_{r(n)}}$ is calculated by selecting 100 failure centers O randomly and we obtain $R_{S_{r(n)}}$.

Result

The evaluation result is shown in Figure 15. The horizontal axis of figure 15 represents the number of nodes and the vertical axis represents efficiency. Compared with Random, Fractal$_{\delta=1.1}$ has high efficiency at any number of nodes. Fractal$_{\delta=1.1}$ holds more short-distance links while increasing the maximum physical link length in proportion to the square root of the number of nodes when the number of nodes increases according to the generation algorithm. On the other hand, Random holds only medium-range links with an average of μ, so even if it is between short-distance nodes, it needs many hops until the nodes reach from/to each other. For this reason, we believe that the difference in hop length for reaching nodes at short distance appears as a difference in efficiency, and the difference is larger as the number of nodes increases.

The result of Figure 15 is calculated using Equation (5) and the reciprocal of the shortest path length of each node pair is averaged. That is, the efficiency indicated as the value of the y axis represents efficiency per node pair. However, when constructing a large-scale virtual network, it is more important to see how much the performance of the entire network improves than to see each node pair. Thus, we consider the sum of the reciprocal of the shortest path length by removing the average part of Equation (5). The result is shown in Figure 16. The horizontal axis of Figure 16 represents the number of nodes and the vertical axis represents the sum of reciprocals of the shortest path length. Looking at Figure 16, as the number of nodes increases, the difference between the sum of reciprocals of hop length expands between Fractal$_{\delta=1.1}$ and Random, In 16384, Fractal$_{\delta=1.1}$ shows a larger value by about 5 million. When there is a difference of 1 in the sum of reciprocals of hop lengths, there is at least a difference of 1 when converted into the sum of hop lengths. Therefore, in the number of nodes 16384, it can be said that Fractal$_{\delta=1.1}$ succeeded in reducing extra communication equivalent to at least 5 million hops in the entire network compared with Random.

The evaluation result on robustness is shown in Figure 17. The horizontal axis of Figure 17 is the number of nodes and the vertical axis is the robustness metric $R_{S_{r(n)}}$. From Figure 17, Fractal$_{\delta=1.1}$ is always lower in value of $R_{S_{r(n)}}$ than Random. In other words, Fractal$_{\delta=1.1}$ is more robust than Random. In addition, the value of Fractal$_{\delta=1.1}$ is more stable. The reason for this result is that Fractal$_{\delta=1.1}$ may be less biased in the structure of the network. Even if you change the scale, Fractal$_{\delta=1.1}$ has the same structure regardless of where you look in the network, so we believe that robustness will be kept constant.

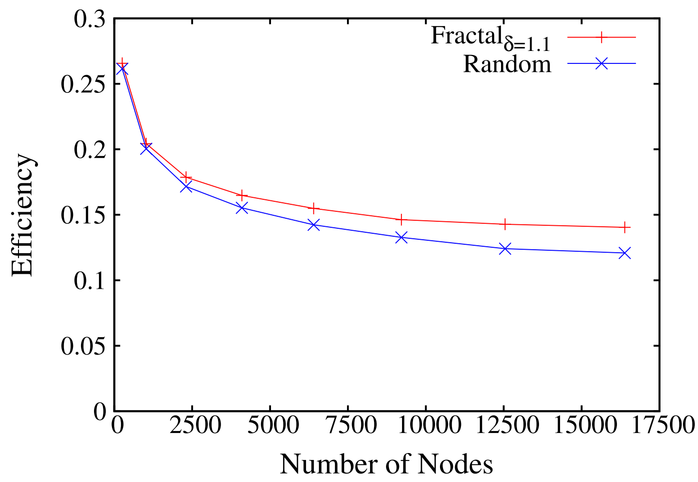


Figure 15: Efficiency of the networks on each scale

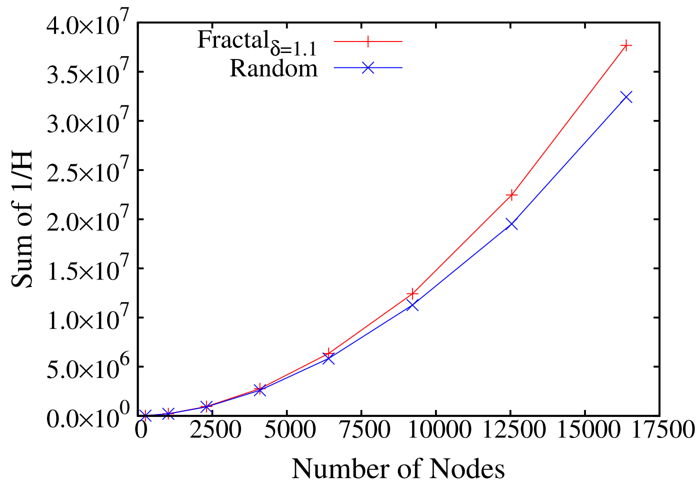


Figure 16: Relationship between the network size and the sum of reciprocals of shortest path length

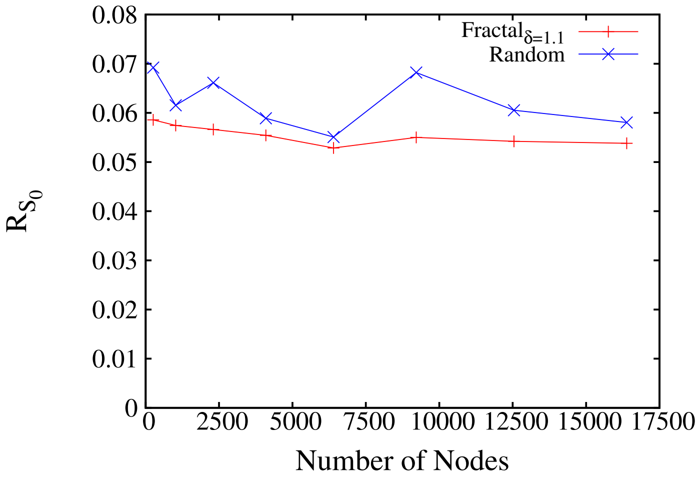


Figure 17: Robustness of the networks on each scale

5 Conclusion

In this thesis, we showed the configuration method for large-scale virtual networks such as IoT networks by focusing on the fractal property and improved the efficiency of the network by controlling the fractal dimension, maintaining high robustness that the fractal structures originally have. Our method generates a fractal virtual network with arbitrary fractal dimension by controlling the exponent of the link length distribution following the power distribution. We evaluated efficiency and robustness of the fractal network in various conditions. First, we revealed that fractal networks are more efficient and more robust than random networks, that is, non-fractal networks because fractal networks have stable reachability between neighbor nodes. Second, as the fractal dimension is higher, long links appear in the network and the efficiency is improved more. Third, even if the scale of the network is changed, the fractal network always shows higher efficiency and higher robustness than the non-fractal network. In conclusion, we showed the guideline for designing a large-scale virtual network with high efficiency and high robustness.

We just show the design method for large-scale virtual networks before services using the networks start to work but the method does not consider constant changes of the network environment such as traffic changes or changes of scale. Therefore, we assume such changes and according to the changes, control and manage the virtual network keeping the fractal property.

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