Evaluation of the degree correlation's impact on information diffusion in modular networks

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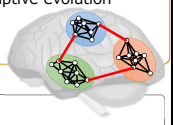
Research Background and Objective

The human brain has some advantages
- Ability to adaptively implement a variety of tasks
- High topological efficiency and robustness while minimizing wiring cost

The topological features of **the brain network** provide several advantages to brain[1,2]

Topological features	Advantages
Small-world properties	→ efficient communication & low cost
Modular structure	→ resilience, adaptive evolution
Assortative mixing within a module	→ ? ? ?
Various mixing patterns between modules	→ ? ? ?



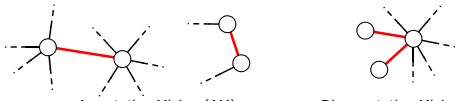
Objective

Revealing the impact of assortativity within and between modules

[1] M. W. Cole, J. R. Reynolds, J. D. Power, G. Repovs,A. Anticevic, and T. S. Braver, "Multi-task connectivity reveals flexible hubs for adaptive task control," Nature neuroscience, vol. 16, no. 9, pp. 1348–1355,2013.
[2] E. Bullmore and O. Sporns, "The economy of brain network organization," Nature Reviews Neuroscience, vol. 13, no. 5, pp. 336–349, May 2012.

Assortativity

The degree correlation between connected nodes
- Assortative Mixing : nodes with **similar** degree tend to be connected
- Disassortative Mixing : nodes with **dissimilar** degree tend to be connected



Assortative Mixing (AM) Disassortative Mixing (DM)

Assortativity coefficient (r) [3] : $r = \frac{1}{\sigma_q^2}\left[\sum_{j,k} jkq(j,k) - \left(\sum_j jq(j)\right)^2\right]$

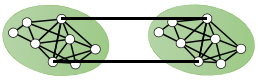
$q(j,k)$: Probability that a randomly chosen edge connects nodes with degree j+1 and k+1
$q(j)$: Probability that degree of a randomly chosen node is j
σ_q : The standard deviation of the remaining degree distribution $q(j)$

[3] M. E. Newman, "Assortative mixing in networks," Physical review letters, vol. 89, no. 20, pp. 1–4, 2002.

Overview of Examination Method

We focus on **two** types of assortativity
- **Assortativity within a module**
  - We evaluate the assortativity coefficient of links within a single module

- **Assortativity between modules**
  - We evaluate the assortativity coefficient of links between two same modules

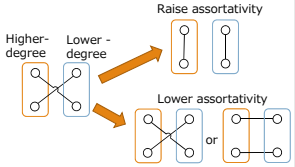


We make networks that have different assortativity and analyze them through some metrics from the point of view of graph theory

How to Make Networks with Different Assortativity?

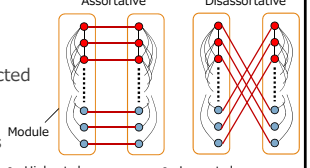
Within a module

1. We make a scale-free network according to the BA model
2. We repeatedly **rewire** two edges
   - This rewiring method changes assortativity without changing the degree distribution



Between modules

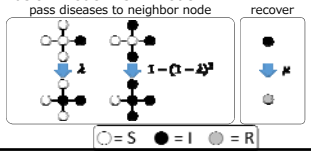
1. We make two same modules
2. *m* highest-degree nodes and *m* lowest-degree nodes are selected in each module
3. We connect two modules in consideration of degree of nodes



● : Highest-degree group ● : Lowest-degree group

Metrics for Evaluation

- **Average hop length**
- **Robustness of connectivity against selective node failure**
  - A decrease tendency of giant component size when node fails in the order of decreasing degree
- **Edge betweenness centrality**
  - The number of the shortest paths that go through an edge in a network
- **Each node's importance on information diffusion**
  - Scale of epidemic when each node is selected as an infection source
  - Information diffusion model : SIR model



Metrics for Evaluation (Single-Module)

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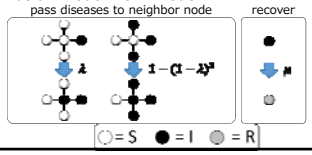
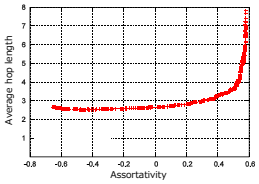
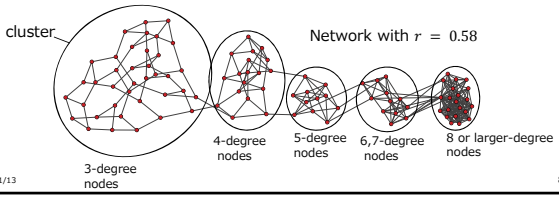


Average Hop Length

As assortativity r becomes larger, the average hop length increases

The average hop length rapidly increases within a range of $r \geq 0.5$
- Strong assortativity makes a module a clustered structure which are concatenated in order of degree

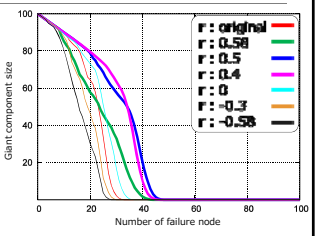


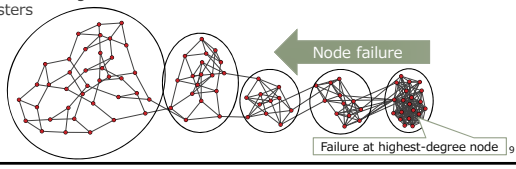


Robustness

An assortative topology is robust against selective node failure
- Node failure occurs from the high-degree side of chain and connectivity is kept

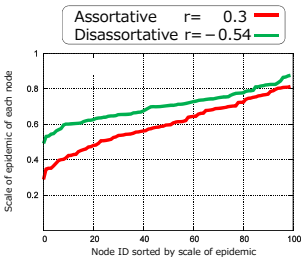
Excessively high assortativity (green line) lowers robustness
- A topology with higher assortativity has a smaller number of edges between clusters

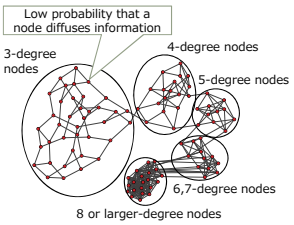




Information Diffusion

We evaluate two networks having the same average hop length but different assortativity





An assortative topology less diffuses information
- As low degree nodes have a low probability of spreading information, in low-degree cluster it is difficult to diffuse information

Metrics for Evaluation (Two-Modules)

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- **Edge betweenness centrality**
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Average Hop Length and Robustness

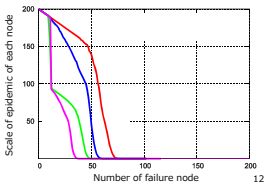
Average hop length

Assortative edges between modules shortens the average hop length
- Assortative edges connect hub nodes through which many shortest paths go

Assortativity		Edges within a module	
		AM	DM
Edges Between modules	AM	3.80 >	3.34
	DM	3.98 >	3.53

Robustness

Assortative edges enhances robustness
- All disassortative edges between modules attach to higher-degree nodes which are removed soon in selective node failures



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	DM		

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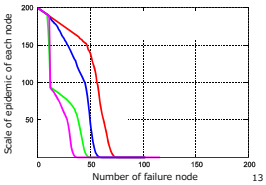

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Assortative edges enhances robustness (red and blue lines)
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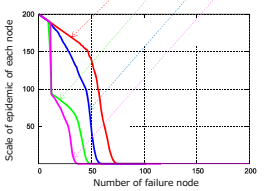
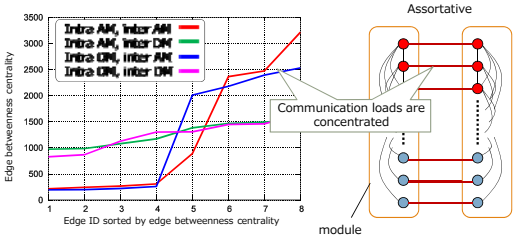


Edge Betweenness Centrality

Disassortative edges distribute communication loads
- When modules are connected assortatively, communication loads are concentrated on edges between high-degree nodes



Impact of Assortativity on Brain Network

Single-module

Mixing patterns of all brain modules are slightly assortative (average 0.23) [4]
- The brain network gets robustness while keeping average hop length low
- In brain network, smaller importance in terms of information diffusion prevents unnecessary information diffusion

Two-modules

Assortativity between modules of the brain network takes various values
- Relations between each pair of modules may be revealed from the point of view of assortativity
  - Assortative : robustness and short average path
  - Disassortative: parallel communication between two modules

[4] P. Hagmann, L. Cammoun, X. Gigandet, R. Meuli, C. J. Honey, V. J. Wedeen, and O. Sporns, "Mapping the structural core of human cerebral cortex," PLoS biology, vol. 6, no. 7, pp. 1479–1493, 2008.

Conclusion and Future Work

Conclusion

We examined the impact of assortativity within and between modules

Single-module
- We revealed that an assortative module has a long average hop length, high robustness, and low information diffusion importance

Two-modules
- We revealed that assortative edges between modules shortens the average hop length, enhances robustness and can be a bottle neck

Future work
- Applying our results to the design of the topology in information networks