Evaluation of the degree correlation's impact on information diffusion in modular networks

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Abstract—The human brain can handle a wide variety of tasks adaptively. In the human brain, anatomical connections make a characteristic network that has high topological efficiency and robustness while minimizing wiring cost. These advantages have been obtained in the process of brain growth and evolution. The human brain network has some topological features as seen in complex networks, such as small-world properties, hierarchical modularity, distinctive degree correlation. However, these topological features are little known to what contributes to brain itself. In this paper, we focus on the hierarchical modular structure and the degree correlation of the human brain network. We show what the benefits of these topological properties are in terms of information diffusion. We also discuss the application of them into information networks.

1. Introduction

The human brain can tackle a large variety of tasks adaptively [1,2]. Recently, due to the advances in neuroimaging techniques, the human brain can be analyzed by much finer spatial resolution. Then, a structural network of the brain represented by the anatomical connections among the region of interest has been studied. Such brain networks are found to have high topological efficiency and robustness while minimizing wiring cost [3,4]. These advantages are considered to have been obtained in the process of human growth and evolution [3].

The brain network has the topological features seen in complex networks–such as small-world properties and hierarchical modular structure [2, 3].Although small-world properties of brain networks are known to lead to their communication efficiency [3, 5], their topological advantages brought by the hierarchical modularity are unrevealed. Opinions are divided: it brings about communication efficiency, robustness, maintaining dynamical activity, and adaptive evolution [2, 5]. It is significant to reveal how these topological properties contribute to the function of brain networks for understanding the human brain and for its technological application.

In this paper, taking the modular structure into account, we focus on the distinctive degree correlation of brain networks, called *assortativity*. Assortativity represents the degree correlations between connected nodes [6]. If a network shows high assortativity (assortative mixing), nodes with similar degree tend to be connected each other. On the other hand, in a network with low assortativity (disassortative mixing), nodes that differ much from their degree are preferentially connected each other. Generally, an assortative-mixing network is robust against selective node failure and accelerates the spreading of information generated from high-degree nodes [6]. Brain networks show modular structure, where densely connected groups of nodes construct modules and they are sparsely connected with each other, and each module presents assortative mixing [5]. However, in previous research, the degree correlation between modules has not been considered.

We focus on the network with modular structure and examine the impact of degree correlation within and between modules in terms of robustness and information spreding. For this purpose, we make networks that have different assortativity and analyze them through some metrics: (1) the average hop length, (2) robustness against selective node failure, in which the highly connected nodes are selectively removed, (3) the edge betweenness centrality [7], and (4) each node's importance on information diffusion [8,9].

First, we focus on a single-module network and reveal the basic properties of an assortative network. Then, we target at a connected two-module network. Although we analyze various networks with respect to degree distributions, in this paper we show the result of the undirected scale-free (SF) network due to space limitations.

2. Method

The assortativity of a network is proposed by Newman as the assortativity coefficient [7]. The assortativity coefficient is calculated from the remaining degree distribution $q(k)$ defined by

$$q(k) = \frac{(k+1)p(k+1)}{\sum_j jp(j)}. \tag{1}$$

The remaining degree distribution is related to the degree distribution $p(k)$, which describes the probability that the degree of a randomly chosen node is k. The remaining degree means the number of edges leaving the vertex other than the one we arrived along. That is, this is less than node's degree by one. Given $q(k)$, joint probability distribution $q(j,k)$ can be introduced, which means the probability that two endpoints of a randomly chosen edge have remaining degree k and j, respectively. Then, assortativity coefficient r is defined as following.

$$r = \frac{1}{\sigma_q^2}\left[\sum_{j,k} jkq(j,k) - \left(\sum_j jq(j)\right)^2\right], \quad (2)$$

where σ_q is the standard deviation of the remaining degree distribution $q(k)$. The range of values that r can take is $[-1, 1]$. It takes positive value when the network is assortative. On the contrary, it takes negative value when the network is disassortative.

Here, we define the degree correlation of the edges between modules. As can be seen in the studies on multiplex networks and interdependent networks [10], degree correlations between networks are often evaluated as well as a single-module network. In this paper, we follow them, that is, we calculate the inter-module degree correlation by using the remaining degrees of endpoint nodes of the edges between modules.

To construct networks with different assortativity, we repeatedly rewire two edges of the original SF network by the following way. First, we randomly choose two edges that do not share the same endpoint. Then, two pair of nodes are rewired so that r approaches the desired value. For example, to raise assortativity, two nodes whose degree is higher than the others are wired. Note that this rewiring method changes assortativity without changing the degree distribution.

As for the assortativity of connections between modules, we connect two networks by the following wiring method. First, in each network, four highest-degree nodes and four lowest-degree nodes are selected. To make connections of networks with assortative mixing, edges connect two nodes that belong to respective module and have a similar degree. By contrast, to connect modules for disassortative mixing, edges connect two nodes that have a dissimilar degree.

Metrics for our evaluation are (1) the average hop length, (2) robustness on selective node failure, (3) the edge betweenness centrality, and (4) each node's importance on information diffusion. Robustness is evaluated by the giant component size when some nodes are removed. Note that a removed node is selected from the highest degree nodes in the remaining network. The edge betweenness centrality is defined as the number of the shortest paths that go through an edge in a network. We use the SIR model to model the diffusion of information. In this model, each node is either susceptible (S), infected (I), or recovered (R). An infected node passes diseases to neighbor nodes with probability β, and recovers itself with probability γ. Consequently, all infected nodes will be recovered. This means, the network is composed of susceptible nodes and/or recovered nodes at the end. The rate of recovered nodes in the network shows the scale of epidemic. By setting each node as the infected node at the initial step, we use this scale ($\mathcal{E}$) as each node's importance on information diffusion.

3. Results

3.1. Single-module network

In this subsection, we investigate a single network (one module) with various assortativity. We use a SF network that has 100 nodes and 295 edges generated by BA model, where a node has at least 3 edges. Then the initial value of r is -0.12. When we try to rewire the edges of this network, assortativity varies within a range of $-0.67 \leq r \leq 0.58$.

Figure 1 shows that as r becomes larger, the average hop length increases. Note that the average hop length rapidly increases within a range of $r \geq 0.5$. In network systems, an increase in the average hop length often means performance degradation.

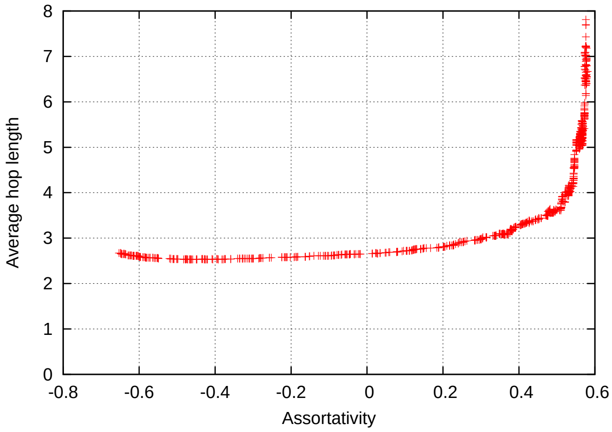

Figure 1: Average hop length in a single-module network

To clarify the reason why the average hop length increases, we show the topology whose assortative is the highest in Figure 2. In this topology, almost all of nodes with the same degree are connected with each other and make a cluster. And clusters are concatenated in order of degree. Generally, a chain-like topology has a longer average hop length (unlike the small-world topology), and therefore, a highly assortative topology has a longer average hop length.

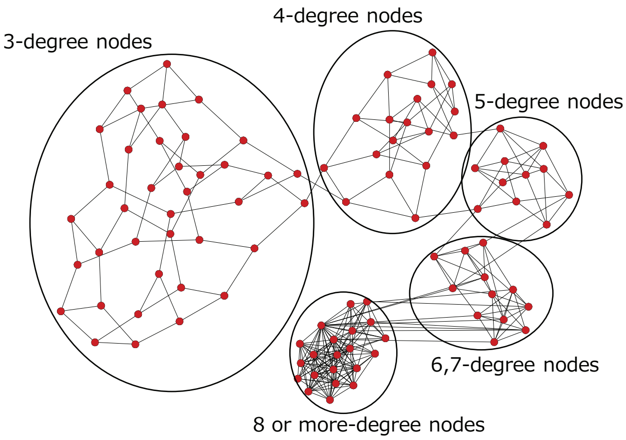

Figure 2: Topology with the maximum assortativity

Figure 3 shows the giant component size of networks with different assortativity when nodes are removed in descending order of degree. A decrease tendency of giant component size depends on the assortativity. As stated above, an assortative topology has some clusters that are

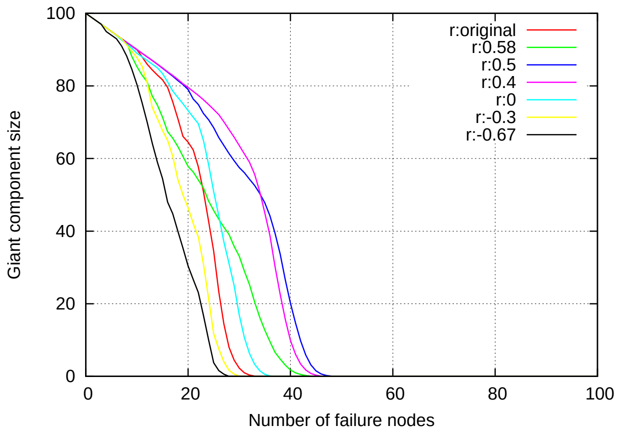

Figure 3: Giant component size in a single-module network

connected with each other like a chain. In our selective node-failure scenario, node failure occurs from the high-degree side of this chain. Therefore, nodes which have a lower degree remain to be connected with each other. This is because an assortative topology is robust against selective node failure. Meanwhile, a topology with higher assortativity has a smaller number of edges between clusters, which decreases robustness. In other words, a rise in assortativity increases the ratio of the number of edges within clusters to the number of edges between clusters.

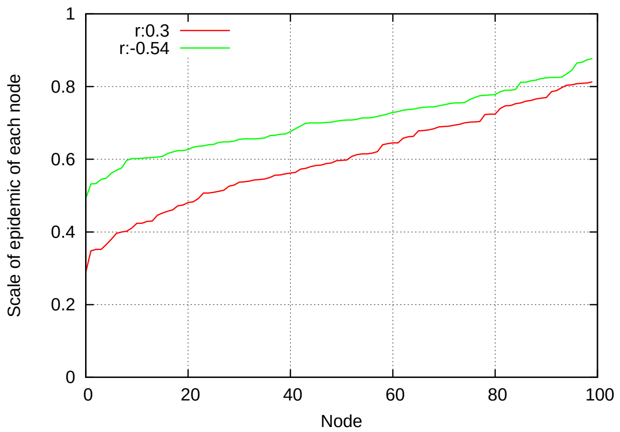

Figure 4: Information-diffusion efficiency in a single-module network

Each node's importance on information diffusion is shown in Figure 4. We evaluate two networks having the same average hop length but different assortativity. In the figure, x-axis shows nodes arranged in the increasing order of $\mathcal{E}$. Figure 4 shows that an assortative topology less diffuses information than a disassortative topology does. In the SIR model, the probability that a node diffuses information to other nodes depends on the number of neighbor nodes. Since low degree nodes have a low probability of diffusing information, low degree nodes is unlikely to diffuse information to the whole network. In an assortative network, low degree nodes have few connections to high degree nodes, which lets the network have low efficiency of information diffusion.

3.2. Two-module network

In this subsection, we investigate the network consisting of two same modules between which 8 edges exist. To construct the two-module network, we use the modules constructed in the previous subsection 3.1 so that AM= 0.3 and DM= −0.3.

Table 1: Average hop length in two-module network

assortativity		edges within a module: AM	edges within a module: DM
edges between modules	AM	3.80	3.34
edges between modules	DM	3.98	3.53

Table 1 shows that high assortativity of edges between modules shortens the average hop length contrary to the result of a single network. This is because edges between modules little change the overall structure of a network. From this perspective, assortativity of inter-module edges is completely different from assortativity of intra-module edges. It is intuitive that the most effective edge connection between modules in terms of the hop count is to link hub nodes because many shortest paths go through hub nodes in the modules. Therefore, assortative edges between modules tend to shorten the average hop length.

Figure 5 shows that high assortativity of inter-module connections enhances robustness. This is a natural result from our selective node-failure scenario. Because all edges between modules in case of disassortative connections attach to higher degree nodes, selective node failures soon remove such nodes (namely, edges between modules), which results in fragmentation.

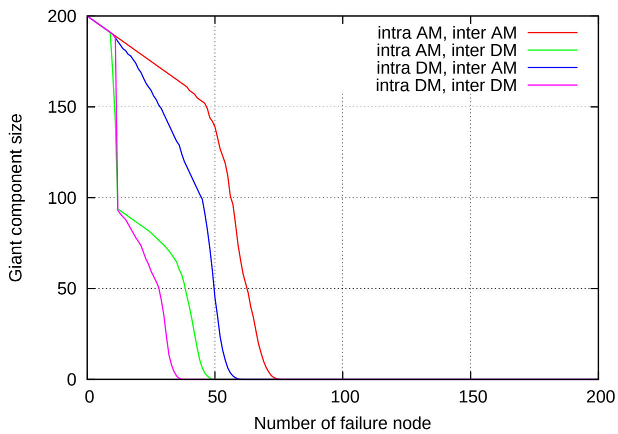

Figure 5: Giant component size in a two-module network

Figure 6 represents the edge betweenness centrality of inter-module edges. The edge betweenness centrality can be considered as communication loads on edges. Accordingly, Figure 6 shows that disassortative edges between modules can distribute communication loads. When modules are connected assortatively, edges between high degree nodes have a significant influence on communication efficiency of the whole network. Owing to this, communication loads are concentrated on such edges. On the other hand, when modules are connected disassortatively, communication loads are distributed over all inter-modules edges because all inter-module edges connect a high degree node and a low degree node.

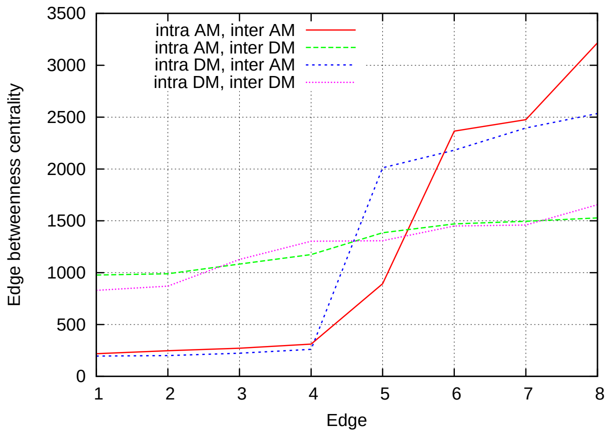

Figure 6: Edge betweenness centrality

4. Discussion

On the assortativity of a single-module network, we revealed that an assortative module has a long average hop length, high robustness, and low information diffusion efficiency. Also, as for the average hop length and robustness, excessively high assortativity gives a bad influence on the network. As for information diffusion, an assortative network has lower performance than a disassortative network, however, from another perspective, assortative network is robust against the spreading of harmful information, e.g., virus infections. Indeed, we analyzed the module of the brain network which is provided in [11]. Then, average assoratativity of modules is 0.23, which is not excessively high. From our results, it probably means that the module in brain networks adjust assortativity for the efficiency and robustness. Additionally, although it is just our speculation, this non-efficiency of information diffusion prevents unnecessary information diffusion. a node in brain network do not diffuse information to indifferent nodes.

Then, we discuss about inter-module assortativity. We revealed the following facts. An assortative edge between modules shortens the average hop length and enhances robustness. A disassortative edge between modules distributes communication loads of inter-module edges. However in both cases, there are several defects. So we should derive a way to connect modules. For example, in the case of connecting modules assortatively, we can adjust link capacity for communication loads.

5. Conclusion

In this paper, we examined the impact of degree correlation within and between modules in terms of robustness and information spreading. On the assortativity of a single module, we revealed that an assortative module has a long average hop length, high robustness, and low information diffusion efficiency. As for the assortativity of inter-module, we revealed that assortative edges between modules shortens the average hop length, enhances robustness and do not distribute communication loads inter-modules. Applying our results to the design of the topology in information networks is our future work.

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References

[1] M. W. Cole, J. R. Reynolds, J. D. Power, G. Repovs, A. Anticevic, and T. S. Braver, “Multi-task connectivity reveals flexible hubs for adaptive task control,” *Nature neuroscience*, vol. 16, no. 9, pp. 1348–1355, 2013.

[2] G. Zamora-López, C. Zhou, and J. Kurths, “Exploring brain function from anatomical connectivity,” *Frontiers in neuroscience*, vol. 5, pp. 1–11, 2011.

[3] E. Bullmore and O. Sporns, “The economy of brain network organization,” *Nature Reviews Neuroscience*, vol. 13, no. 5, pp. 336–349, May 2012.

[4] E. Bullmore and O. Sporns, “Complex brain networks: graph theoretical analysis of structural and functional systems,” *Nature Reviews Neuroscience*, vol. 10, no. 3, pp. 186–198, 2009.

[5] D. Samu, A. K. Seth, and T. Nowotny, “Influence of wiring cost on the large-scale architecture of human cortical connectivity,” *PLoS Computtational Biology*, vol. 10, pp. 1 – 24, 2014.

[6] M. E. Newman, “Assortative mixing in networks,” *Physical review letters*, vol. 89, no. 20, pp. 1–4, 2002.

[7] U. Brandes, “On variants of shortest-path betweenness centrality and their generic computation,” *Social Networks*, vol. 30, no. 2, pp. 136–145, 2008.

[8] K. Klemm, M. Á. Serrano, V. M. Eguíluz, and M. San Miguel, “A measure of individual role in collective dynamics,” *Scientific reports*, vol. 2, pp. 1–7, 2012.

[9] S. Bansal, B. T. Grenfell, and L. A. Meyers, “When individual behaviour matters: homogeneous and network models in epidemiology,” *Journal of the Royal Society Interface*, vol. 4, no. 16, pp. 879–891, 2007.

[10] V. Nicosia and V. Latora, “Measuring and modelling correlations in multiplex networks,” *arXiv preprint arXiv:1403.1546*, vol. 2, pp. 1–22, 2014.

[11] P. Hagmann, L. Cammoun, X. Gigandet, R. Meuli, C. J. Honey, V. J. Wedeen, and O. Sporns, “Mapping the structural core of human cerebral cortex,” *PLoS biology*, vol. 6, no. 7, pp. 1479–1493, 2008.