Hierarchical optimal control method for controlling self-organized networks with light-weight cost

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***Abstract*—Self-organization has potential for high scalability, adaptability, flexibility, and robustness, which are vital features for realizing future networks. Convergence of self-organizing control, however, is comparatively slow in some practical applications. It is therefore important to enhance convergence of self-organizing controls without sacrificing the above advantages. Controlled self-organization is one key idea for that, which introduces an external controller into self-organizing systems to guide them to a desired state. We previously designed an external controller that provided the optimal input for fast convergence, however, it suffered from scalability issues. In this paper, we propose a hierarchical control system where a network is partitioned into some sub-networks, sub-controllers manage respective sub-networks, and the top-level deals with the global network stability. The proposed system achieves fast convergence speed with low computational and communication cost.**

***Keywords*-Controlled self-organization, hierarchical control, potential-based routing, fast convergence**

I. INTRODUCTION

Self-organization is a natural phenomenon in distributed systems [1]. In a self-organizing system, a global behavior or pattern emerges from local interactions among components in a bottom-up manner. For future large-scale and complex networks, many researchers focus on the idea of controlled (guided, managed) self-organization, where the self-organizing system is controlled through some constraints while retaining strengths of self-organization such as scalability, adaptability, robustness and flexibility [2].

We previously introduced an external controller having an optimal feedback mechanism to self-organizing systems in [3] for controlling self-organization to converge faster. The external controller collects information regarding the network such as node states and the network topology via a partial set of nodes directly monitored by the controller, estimates system dynamics using a mathematical model that describes the network dynamics, and then determines optimal control inputs based on robust control theory [4]. We showed that the mechanism improves the convergence speed of self-organization in [3]. However, especially in large-scale networks, it is difficult for "one" centralized controller to collect detailed network information and calculate optimal feedback inputs. Doing so requires considerable costs and, even worse, loses the scalability of self-organizing systems.

In this paper, we propose a hierarchical optimal feedback mechanism to control self-organizing network systems without losing the scalability of self-organization. With our proposal, a network is partitioned into several sub-networks, which are controlled in a hierarchical manner by two types of controllers, a *top-level central-controller* and several *sub-controllers*. Each sub-controller monitors a different sub-network, i.e., only a part of the entire network, and provides an optimal feedback input to the sub-network so that fast convergence can be achieved. Then, communication cost for collecting the network information would be reduced. In detail, given N_{sub} nodes in a sub-network, the sub-controller corresponding to the sub-network needs $O({N_{sub}}^2)$ for calculating an optimal feedback input to the sub-network. The computational cost of each sub-controller is much smaller than $O(N^2)$ of the external controller proposed in [3] where N ($> N_{sub}$) is the number of nodes in the entire network. Therefore, owing to sub-network partitioning, self-organizing systems can be controlled with light-weight cost even if the network size is large. By contrast, if each sub-controller determines an optimal input selfishly, interactions among sub-networks may cause network instability. So, the role of a central controller is to guide sub-networks to achieve the identical global optimality. A central controller collects information from sub-controllers, estimates degrees of interactions among sub-networks and then returns feedback inputs to sub-controllers. Note that, given the information of the network topology, we need the computation cost $O(N^2)$ for designing the central controller, whereas that for designing the external controller in [3] is $O(N^3)$.

We consider potential-based routing, which is a self-

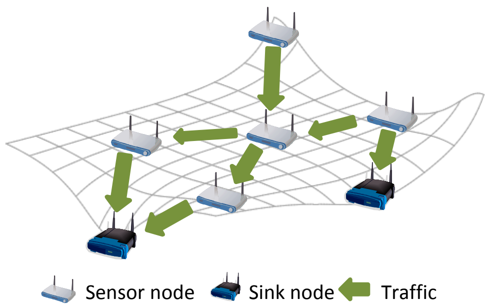


Figure 1. Potential-based routing

organizing routing mechanism as well as in [3]. We introduce the hierarchical optimal feedback mechanism to it and show the fast convergence speed after environmental changes. The optimality of our feedback mechanism is analytically guaranteed in synchronous systems, but not in asynchronous systems. Then, we assume a wireless sensor network, where nodes behave asynchronously, to show the practicality of our proposal. Through computer simulation, we show that our hierarchical optimal feedback mechanism can enhance the convergence speed of self-organizing systems at fairly low cost.

II. Potential-based Routing

Potential-based routing is a self-organizing routing mechanism being active in the fields of wireless sensor networks, mobile ad-hoc networks, and information centric networks [5], [6]. Here we assume that potential-based routing is used in wireless sensor networks. In potential-based routing, each node has a scalar value called its *potential*, and data packets are forwarded to a neighbor whose potential is smaller than the forwarder's. In wireless sensor networks, data packets are generally sent to a sink node, and the fewer hops from the sink node a node is, the lower the potential value assigned to the node. The simple forwarding rule "forward data to a neighboring node with lower potential" can therefore result in data packet collection toward sink nodes, as illustrated in Fig. 1. Potential-based routing has high scalability because each node uses only local information for calculating potentials and a local rule for forwarding data. Furthermore, it can achieve the load balancing and consequently the network lifetime improvement by calculating potential using information such as flow rates, queue length, or remaining energy.

A. Potential Field Construction

Sheikhattar and Kalantari [6] focused on the convergence of potential-based routing and achieved enhancement of the potential convergence speed. They proposed a potential calculation method based not only on current potentials but also on last potentials to accelerate potential convergence. Node n's potential at time t, $\theta_n(t)$, is calculated by (1).

$$\begin{aligned}\theta_n(t+1) =& (\alpha+1)\theta_n(t) - \alpha\theta_n(t-1) \\ &+ \beta\sigma_n\left(\sum_{k\in\mathcal{N}_b(n)}\{\theta_k(t)-\theta_n(t)\} + f_n(t)\right),\end{aligned} \tag{1}$$

where $\mathcal{N}_b(n)$ is a set of node n's neighbors, and α is a parameter that determines weights of current and last potential values when calculating the next potential. Larger α means that the weight of the last potential value is larger and therefore the system becomes less subject to current noise, though the convergence speed is slower. Parameter β determines the influence amount of neighbor node potentials. σ_n is defined as $1/|\mathcal{N}_b(n)|$, and $f_n(t)$ corresponds to the flow rate of node n at time t. If $f_n(t)$ is a positive value it means the data generation rate of node n, whereas if $f_n(t)$ is negative it means the rate of data packets delivered to node n. For sink node n, $f_n(t)$ corresponds to targeted flow rate and given by the network manager. If the flow conservation constraint is upheld, that is, $\sum_{n\in\{1,\cdots,N\}} f_n(t) = 0$, a potential field is constructed so that actual rates of data packets delivered to nodes satisfy given flow rates, i.e., all gradients, which are potential differences between next hop nodes, correspond the appropriate flow rates.

The convergence speed based on (1) is faster than simple Jacobi iterations (such as our previous work [5]), but still takes a long time to converge due to its calculation being based only on local information.

B. Routing

If a node has a data packet, it forwards it according to potential values of itself and its neighbors. In our potential-based routing, when a sensor node generates or receives a data packet, it probabilistically selects a next node that is assigned a lower potential value than itself, and the packet eventually arrives at a sink node. Specifically, a next-hop node is selected proportionally with potential values, that is, the probability $p_{i\to j}(t)$ that sensor node i selects a neighbor node j as the next-hop node for a data packet at time t is given by

$$p_{i\to j}(t) = \begin{cases} \frac{\theta_i(t)-\theta_j(t)}{\sum_{k\in\mathcal{N}_l(i)}\{\theta_i(t)-\theta_k(t)\}}, & \text{if } j\in\mathcal{N}_l(i) \\ 0, & \text{otherwise} \end{cases},$$

where $\mathcal{N}_l(i)$ is the set of node i's neighbor nodes that are assigned lower potential values than node i.

III. Potential-based Routing with Hierarchical Optimal Feedback

In this section, we describe a model of the network dynamics and explain our hierarchical optimal control scheme.

A. Overview

Proposed hierarchical optimal feedback is shown in Fig. 2 where there are three layers, i.e., a layer of physical nodes, one of sub-controllers and one of a central controller. A network is partitioned into several sub-networks and each

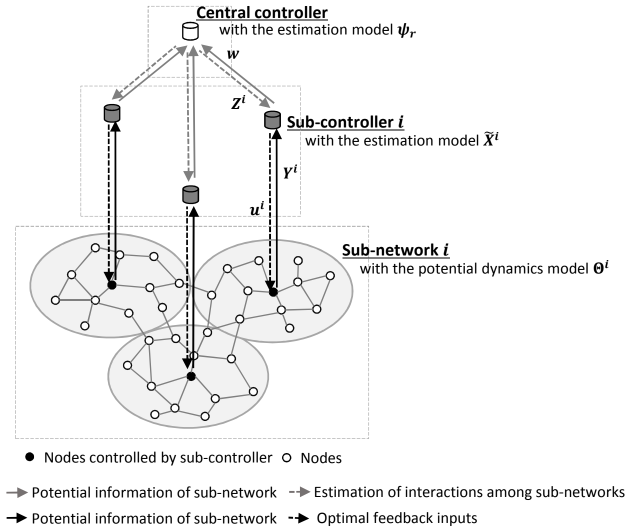


Figure 2. Potential-based routing with a hierarchical optimal control scheme, where sub-controllers monitor/control their own area, and the central controller collects potential values from sub-controllers and provide feedback inputs to them periodically.

sub-network is connected to a sub-controller via a part of nodes that belong to the sub-network. We assume that each sub-network has at least one sink node and a sub-controller is connected to a sink node.

A *sub-controller* monitors information of its corresponding sub-network, in particular potential values of a partial set of nodes, which we call *observable nodes*, in the sub-network. The sub-controller then returns suitable control inputs to a partial set of nodes, which we call *controllable nodes*, in the sub-network, for accelerating convergence of the potential distribution of the sub-network toward the target potential distribution. In this paper, target potentials are estimates of converged potential values derived from collected information, specified in Subsection III-B. When receiving control inputs from the sub-controller, controllable nodes calculate their potential, which changes potential amounts over the entire network through local interactions among nodes ((2) in Subsection III-B). Note that we assume that sub-controllers and sink nodes are power-supplied so that these sink nodes can have a directly reliable connection with sensor nodes and sub-controllers at all times. Therefore, in our proposal, sub-controllers monitor sub-network information and provide control inputs via them.

The role of a central controller is to estimate interactions among sub-networks and to provide feedback inputs to sub-controllers so that the whole network reaches the identical targeted state. For that purpose, the central controller collects the network information from sub-controllers, i.e., potential values of a partial set of nodes, and then returns suitable feedback inputs to them. Without the central controller, that is, ignoring interactions among sub-networks, not only the global optimality of potential distribution is not guaranteed, but also network instability may be caused because each sub-controller cannot directly observe any information of sub-networks except for its corresponding one.

Each node updates its potential value in a self-organizing manner as mentioned in Section II with and without control feedback. Therefore, our control method can enhance the convergence speed of potentials retaining advantages of self-organizing systems. Moreover, if the central controller or a sub-controller fails, each node behaves individually and automatically so that potential convergence can be achieved eventually.

In our proposal, the area over which each sub-controller monitors potential values is limited to a partial set of nodes in its corresponding sub-network to reduce the communication costs. Of course, the sub-controllers cannot directly get node potentials outside the area, but the central controller estimates them by utilizing the potential dynamics model, which describes potential changes. Note that information of the sub-network topology is needed for designing the sub-controller, as is flow rates of nodes in the sub-network for calculating target potential values. Such information is difficult to estimate, but is reported to the sub-controllers only when it changes because we assume that changing frequencies of the network topology and flow rates are much lower than those of potentials. This assumption is feasible because the potential convergence is generally achieved as a result of the iterative behavior (nodes' potential updates and the controller's feedback) so that frequencies of potential updates and controls need to be much higher than those of changes the network topology and flow rates.

B. Dynamics of Sub-Networks

Let the dynamics of potentials be given by a deterministic discrete-time model. In our proposal, each potential is updated as well as in [6], but controllable nodes additionally receive feedback inputs by each sub-controller. Node n updates its potential at time t by (2).

$$\theta_n(t+1) = (\alpha+1)\theta_n(t) - \alpha\theta_n(t-1) + \beta\sigma_n \left(\sum_{k\in\mathcal{N}_b(n)} \{\theta_k(t) - \theta_n(t)\} + f_n \right) + \eta_n(t), \qquad (2)$$

where η_n represents a feedback input received from the sub-controller. If node n does not receive any feedback input directly from the sub-controller, then $\eta_n(t) = 0$. Sub-controllers collect and estimate the node potentials of each sub-network, and provide to the sub-network feedback inputs $\boldsymbol{u}(t) = [\eta_1(t)\ \eta_2(t)\ \cdots\ \eta_{N_{ctrl}}(t)]^T$ (where N_{ctrl} denotes the number of nodes that receive feedback from the controller), as described later. Node potentials can converge faster than in the non-control scheme described by (1) where each node updates its potential based only on local interactions with neighbors. In [6], σ_n is set to $1/|\mathcal{N}_b(n)|$, but this value may lead to oscillation of potentials in some situations since (4) has no solution (after-mentioned). In this paper we therefore set σ_n to the constant value σ

$(0 < \sigma < 1)$ for all n $(\in \{1, 2, \cdots, N\})$.

Then, we describe potential dynamics of sub-networks. We define $\mathcal{S}$ as a set of sub-networks. Sub-network i $(\in \mathcal{S})$ includes N^i nodes. Potential values of nodes in sub-network i are described as a vector $\boldsymbol{\Theta}^i(t) = \left[\boldsymbol{\theta}_1^i(t)^T\ \boldsymbol{\theta}_2^i(t)^T\ \cdots\ \boldsymbol{\theta}_{N^i}^i(t)^T\right]^T$ using $\boldsymbol{\theta}_n(t) = [\theta_n(t)\ \theta_n(t+1)]^T$. The potential dynamics of sub-network i is given by (3) using flow matrix $\boldsymbol{F}^i$ and control inputs $\boldsymbol{u}^i$.

$$\boldsymbol{\Theta}^i(t+1) = \boldsymbol{A}^i\boldsymbol{\Theta}^i(t) + \sum_{j\in\mathcal{S}-\{i\}} \boldsymbol{A}^{i,j}\boldsymbol{\Theta}^j(t) + \left(\beta\sigma\boldsymbol{F}^i + \boldsymbol{u}^i(t)\right)\otimes\boldsymbol{M}_0, \quad (3)$$

where $\boldsymbol{A}^i$, $\boldsymbol{A}^{i,j}$ are matrices that describe interactions within sub-network i, and between sub-network i and j, respectively [1].

Under these dynamics, potentials $\boldsymbol{\Theta}^i$ consequently converge to $\bar{\boldsymbol{\Theta}}^i$, which is given by a solution of

$$(\boldsymbol{A}^i - \boldsymbol{I}_{2N^i\times 2N^i})\bar{\boldsymbol{\Theta}}^i + \sum_{j\in\mathcal{S}-\{i\}} \boldsymbol{A}^{i,j}\bar{\boldsymbol{\Theta}}^j = -\beta\sigma\boldsymbol{F}^i\otimes\boldsymbol{M}_0. \quad (4)$$

In the evaluation of Section IV, we use $\bar{\boldsymbol{\Theta}} = \left[\bar{\boldsymbol{\Theta}}^{1T}\ \cdots\ \bar{\boldsymbol{\Theta}}^{|\mathcal{S}|T}\right]^T$ as a target potential distribution.

C. Optimal Control of Sub-networks by sub-controllers

We next explain the sub-controller dynamics. For all i $(\in \mathcal{S})$, sub-network i is connected to sub-controller i, and sub-controller i directly monitors potentials $\boldsymbol{Y}^i$ of observable nodes in sub-network i. $\boldsymbol{Y}^i(t)$ is a $2N_{obs}^i$-dim vector, which is given by $\boldsymbol{Y}^i(t) = \boldsymbol{C}^i\boldsymbol{X}^i(t)$ [2] using $\boldsymbol{X}^i(t) = \bar{\boldsymbol{\Theta}} - \boldsymbol{\Theta}^i(t)$ where N_{obs}^i is the number of observable nodes. Note that $\boldsymbol{Y}^i$ shows how different potentials of observable nodes are from their target values. Then, sub-controller i estimates $\boldsymbol{X}^i(t)$ from observable information $\boldsymbol{Y}^i$. $\tilde{\boldsymbol{X}}^i(t)$ is $2N^i$-dim vector and given by (5) and (6) [3].

$$\tilde{\boldsymbol{X}}^i(t+1) = \boldsymbol{A}^i\tilde{\boldsymbol{X}}^i(t) + \boldsymbol{B}^i\boldsymbol{u}^i(t) + \boldsymbol{Q}^i\left(\boldsymbol{Y}^i(t) - \tilde{\boldsymbol{Y}}^i(t)\right) + \boldsymbol{Z}^i(t), \quad (5)$$

$$\tilde{\boldsymbol{Y}}^i(t) = \boldsymbol{C}^i\tilde{\boldsymbol{X}}^i(t). \quad (6)$$

where $\boldsymbol{Z}^i$ means interactions among sub-networks. Sub-controller i receives $\boldsymbol{Z}^i$ from the central controller and calculates a feedback input that accelerates the convergence speed of potentials, which we explain in Subsection III-D. If $\tilde{\boldsymbol{X}}^i(t)$ is close to 0, potentials are estimated to be close to their targets. Then, the feedback input $\boldsymbol{u}^i(t)$ is calculated according to (7) [4].

$$\boldsymbol{u}^i(t) = -\boldsymbol{V}^i\tilde{\boldsymbol{X}}^i(t), \quad (7)$$

D. Estimation of Interactions among Sub-networks by the Central Controller

1) Design of Central Controller: Here, we explain how the central controller estimates interactions among sub-networks. The central controller collects potential information $\boldsymbol{w}(t)$ from sub-controllers for estimating interactions among sub-networks. $\boldsymbol{w}(t)$ is a $2N_{obs}$-dim vector, with N_{obs} being the number of nodes whose potentials are collected by the central controller, and given by $\boldsymbol{w}(t) = \boldsymbol{W}\boldsymbol{X}(t)$ [5]. Note that observable nodes for the sub-controllers and nodes whose potential information is collected by the central controller are not necessary the same, i.e., $\boldsymbol{w}(t)$ does not need to be equal to $\left[\boldsymbol{Y}^1(t)^T\ \boldsymbol{Y}^2(t)^T\ \cdots\ \boldsymbol{Y}^{|\mathcal{S}|}(t)^T\right]^T$.

Then, the central controller estimates strengths $\boldsymbol{Z}(t)$ of interactions among sub-networks by (9) and (10) [6] [7].

[1] $\boldsymbol{A}^i$, $\boldsymbol{A}^{i,j}$ are given by

$$\boldsymbol{A}^i = \boldsymbol{I}_{N^i\times N^i}\otimes\boldsymbol{A}_1 + (\boldsymbol{L}_{intra}^i - \boldsymbol{G}^i)\otimes\boldsymbol{A}_0,$$
$$\boldsymbol{A}^{i,j} = \boldsymbol{L}_{inter}^{i,j}\otimes\boldsymbol{A}_0,$$

where $(N^i\times N^i)$-matrix $\boldsymbol{L}_{intra}^i$ corresponds to the adjacent matrix of sub-network i. $\boldsymbol{L}_{inter}^{i,j}$ is a $(N^i\times N^j)$-matrix, which describes links connecting between subnetwork i and j. The element $l_{inter}^{i,j}(n,m) \in \{0,1\}$ of $\boldsymbol{L}_{inter}^{i,j}$ is 1, if and only if there is a link between node n of sub-network i and node m of sub-network j. $(N^i\times N^i)$-matrix $\boldsymbol{G}^i$ describes degrees of nodes that belongs to sub-network i, and the element $g^i(n,n)$ of $\boldsymbol{G}^i$ corresponds to the number of links one of whose edges is node n. $\boldsymbol{I}_{N\times N}$ is the identity matrix of $N\times N$. $\boldsymbol{A}_1$, $\boldsymbol{A}_0$ and $\boldsymbol{M}_0$ are given by

$$\boldsymbol{A}_1 = \begin{bmatrix} 0 & 1 \\ -\alpha & \alpha+1 \end{bmatrix}, \boldsymbol{A}_0 = \begin{bmatrix} 0 & 0 \\ 0 & \beta\sigma \end{bmatrix}, \boldsymbol{M}_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

[2] An $(2N_{obs}^i\times 2N^i)$-matrix $\boldsymbol{C}^i$ determines observable nodes. The element $c^i(n,m)\in\{0,1\}$ of $\boldsymbol{C}^i$ is 1 if and only if sub-controller i monitors the potential value of node m as the n-th element of $\boldsymbol{Y}^i$. If more nodes are monitored by sub-controller i, i.e., $\boldsymbol{C}^i$ is denser, it can estimates potential values of nodes in sub-network i more precisely while the communication overhead for collecting information of observable nodes becomes much larger.

[3] We need to select $\boldsymbol{Q}^i$ which satisfies "$\boldsymbol{A}^i - \boldsymbol{Q}^i\boldsymbol{C}^i$ is stable," so that potentials converge. The matrix $\boldsymbol{B}^i$ is given by

$$\boldsymbol{B}^i = \boldsymbol{E}^i\otimes\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

The $(N^i\times N_{ctrl}^i)$-matrix $\boldsymbol{E}^i$ specifies the controllable node of sub-network i, that is, the element $e^i(n,m)\in\{0,1\}$ of $\boldsymbol{E}^i$ is 1 if and only if node n receives the m-th element of $\boldsymbol{u}^i(t)$ as control input $\eta_n(t)$. With the larger number of controllable nodes, i.e., with denser $\boldsymbol{E}^i$, the influence of the optimal control is larger so that the convergence speed should be faster although significant properties originating from self-organization such as scalability and adaptability could be lost with too many controllable nodes.

[4] $\boldsymbol{V}^i$ is the optimal gain matrix that minimizes the quadratic cost function $H^i(\boldsymbol{u}^i) = \sum_{n=1}^{\infty}\left(||\tilde{\boldsymbol{X}}^i(n)|| + r||\boldsymbol{u}^i(n)||\right)$ for the system. r is a parameter that regulates the trade-off between the convergence speed and stability of potentials. With lower r, the convergence speed of potentials is faster but potentials change more largely at one time because the sub-controller is allowed to provide larger $\boldsymbol{u}^i(t)$.

[5] $\boldsymbol{X}(t)$ describes the potential dynamics of all nodes and $\boldsymbol{X}(t)$ is defined as $\left[\boldsymbol{X}^1(t)^T\ \boldsymbol{X}^2(t)^T\ \cdots\ \boldsymbol{X}^{|\mathcal{S}|}(t)^T\right]^T$. The dynamics of the entire network is given by

$$\boldsymbol{X}(t+1) = \boldsymbol{A}\boldsymbol{X}(t) + \boldsymbol{B}\boldsymbol{u}(t), \quad (8)$$

where

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}^1 & \boldsymbol{A}^{1,2} & \dots & \boldsymbol{A}^{1,|\mathcal{S}|} \\ \boldsymbol{A}^{2,1} & \boldsymbol{A}^2 & \dots & \boldsymbol{A}^{2,|\mathcal{S}|} \\ \vdots & & & \vdots \\ \boldsymbol{A}^{|\mathcal{S}|,1} & \boldsymbol{A}^{|\mathcal{S}|,2} & \dots & \boldsymbol{A}^{|\mathcal{S}|} \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} \boldsymbol{B}^1 \\ \boldsymbol{B}^2 \\ \vdots \\ \boldsymbol{B}^{|\mathcal{S}|} \end{bmatrix}.$$

$\boldsymbol{W}$ is a $(2N_{obs}\times 2N)$-matrix that determines nodes whose potential are collected by the central controller in the same manner as $\boldsymbol{C}^i$.

[6] $\boldsymbol{J}$, $\boldsymbol{K}$, $\boldsymbol{O}$ and $\boldsymbol{T}$ are given by

$$\boldsymbol{\psi}(t+1) = \boldsymbol{J}\boldsymbol{\psi}(t) + \boldsymbol{K}\boldsymbol{u}(t) + \boldsymbol{O}\boldsymbol{w}(t), \quad (9)$$

$$\boldsymbol{Z}(t) = \boldsymbol{T}\boldsymbol{\psi}(t), \quad (10)$$

where $2N$-dim vector $\boldsymbol{\psi}$ is describes an estimation model for the central controller, with which the central controller estimates the interactions among sub-networks, and $\boldsymbol{Z}(t)$, which corresponds to $\left[\boldsymbol{Z}^1(t)^T \; \boldsymbol{Z}^2(t)^T \; \cdots \; \boldsymbol{Z}^{|\mathcal{S}|}(t)^T\right]^T$, is provided to each sub-controller by the central controller.

2) Model Reduction for Central Controller: With the estimation model described by (9) and (10), which has $2N$ state variables, strength $\boldsymbol{Z}(t)$ are calculated with $O(N^2)$. In other words, the computational cost is extremely large in large-scale networks. In our proposal, therefore, the central controller uses a reduced-order model [7] which has less state variables for estimating interactions among sub-networks with a smaller computational cost. In the reduced-order model, the estimation model is expressed as an $(h \times 1)$-vector $\boldsymbol{\psi}_r(t)$ whose elements are linear transformations of the original model $\boldsymbol{\psi}(t)$, and the reduced-order model is given by (11) and (12).

$$\boldsymbol{\psi}_r(t+1) = \boldsymbol{J}_r\boldsymbol{\psi}_r(t) + \boldsymbol{K}_r\boldsymbol{u}(t) + \boldsymbol{O}_r\boldsymbol{w}(t), \quad (11)$$

$$\boldsymbol{Z}_r(t) = \boldsymbol{T}_r\boldsymbol{\psi}_r(t). \quad (12)$$

Here, $2N$-dim vector $\boldsymbol{Z}_r(t)$ corresponds to feedback inputs provided to sub-controllers by the central controller with the reduced-order model. We need to choose matrices $\boldsymbol{J}_r$, $\boldsymbol{K}_r$, $\boldsymbol{O}_r$ and $\boldsymbol{T}_r$ of compatible dimensions such that $\boldsymbol{Z}_r(t) \approx \boldsymbol{Z}(t)$ for all input $\boldsymbol{u}(t)$, $\boldsymbol{w}(t)$.

A reduced-order model can be described with a constant number h of state variables and the computational cost can be reduced using it. With a reduced-order model, feedback inputs $\boldsymbol{Z}_r(t)$ are calculated with $O(h^2)$. In general, a model that has more state variables allows the controller estimate more correctly, but the computational cost is larger. In contrast, the computational cost is smaller but the estimation error can be larger in a model that has fewer state variables. Therefore, h needs to be properly determined in accordance with the requirements or system properties.

IV. Performance Evaluation

In this section, we evaluate the convergence speed of potentials comparing our proposal with the non-control

$$\boldsymbol{J} = \boldsymbol{A} - \boldsymbol{Q}\boldsymbol{W},\ \boldsymbol{K} = \boldsymbol{B},\ \boldsymbol{O} = \boldsymbol{Q},$$

$$\boldsymbol{T} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{A}^{1,2} & \cdots & \boldsymbol{A}^{1,|\mathcal{S}|} \\ \boldsymbol{A}^{2,1} & \boldsymbol{0} & \cdots & \boldsymbol{A}^{2,|\mathcal{S}|} \\ \vdots & & & \vdots \\ \boldsymbol{A}^{|\mathcal{S}|,1} & \boldsymbol{A}^{|\mathcal{S}|,2} & \cdots & \boldsymbol{0} \end{bmatrix}.$$

$\boldsymbol{Q}$ is a $(2N \times 2N_{obs})$-matrix which satisfies "$\boldsymbol{A} - \boldsymbol{Q}\boldsymbol{W}$ is stable." $\boldsymbol{0}$ is a zero matrix.

[7]Many researchers have studied a variety of methods to approximate a model with a reduced-order model to control large, complex systems [8]. In our proposal, we approximate the original model based on a '*balanced realization*' that is highly compatible with the model expressed in the state space representation [4], [8]. In model reduction, a reduced-order model needs to have the same response characteristics as the original model for the accurate estimation of potentials. Here, response characteristics mean the effectiveness of inputs to the system.

scheme proposed in [6] to show that hierarchical optimal feedback enhances the convergence.

In simulation experiment, for the network simulator we use an event-driven packet-level simulator written in Visual C++ that calls MATLAB functions *dlqr* to design the optimal controller and *balred* to obtain a reduced-order model.

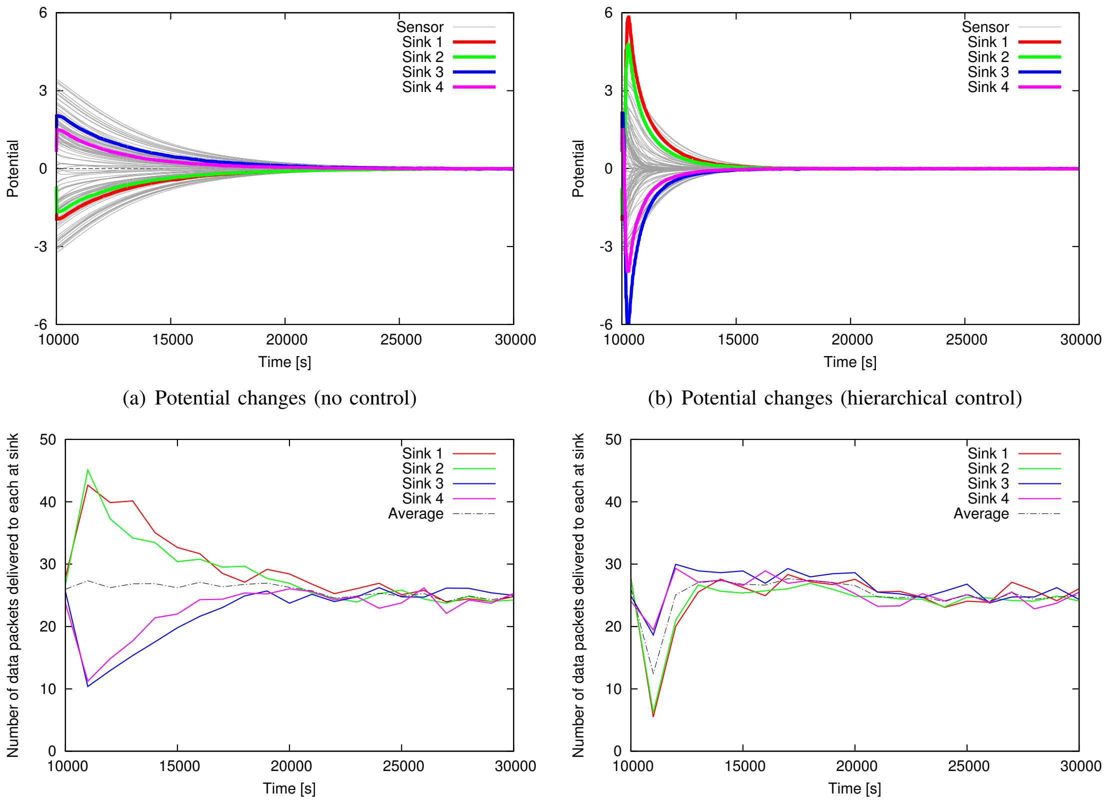
A. Simulation Settings

We evaluate changes of potentials and the number of data packets delivered to each sink node after traffic changes. The network model with 104 nodes (including 4 sink nodes) is used for this evaluation. 4 sink nodes are directly connected to 4 different sub-controllers, and sub-controllers are connected to the central controller. The central controller provides feedback inputs to sub-controllers at interval T_{cc}, sub-controllers provide feedback inputs to each sink nodes at interval T_{pc}, and, each node updates its potential value at interval T_p. Typically, $T_{cc} = T_{pc} = T_p$ for matching with the potential dynamics described by (2) and (5). In this evaluation, we set T_{cc}, T_{pc} and T_p to 50 s. Each sub-controller observes all nodes of the corresponding sub-network, and the central controller collects potential information of nodes within 2 hops from each sink node. In this evaluation, we assume that controllers can collect the potential information with no communication delay because the main target of this evaluation is to reveal the performance and the property of the hierarchical control scheme. The model order h of the central controller is set to 20.

At the beginning of the simulation, potential values of all nodes, including sensor nodes and sink nodes, are initialized to 0. At 1,000 s data packets begin to be generated at sensor nodes according to the Poisson process with their flow rates. At 10,000 s from the beginning of the simulation, data packet generation rates of sensor nodes are changed in order to examine the convergence speeds of potentials and traffic flows. Data packet generation rates are initially set to be 0.0005 packets/s for sensor nodes in the half of the network, and 0.0015 packets/s for sensor nodes in the other half. After traffic changes at 10,000 s, data packet generation rates are increased to 0.0015 packets/s for the former half sensor nodes and decreased to 0.0005 packets/s for the latter half nodes. The average data generation rate of a node of 0.001 packets/s corresponds to $\bar{f}_n = 1$, so before traffic changes the flow rate vector $\boldsymbol{F} = \left[\bar{f}_1 \cdots \bar{f}_N\right]^T$ is given by $\boldsymbol{F} = [0.5 \cdots 0.5\ 1.5 \cdots 1.5 - 25 \cdots - 25]^T$. Similarly, after the traffic changes, $\boldsymbol{F}$ is given by $\boldsymbol{F} = [1.5 \cdots 1.5\ 0.5 \cdots 0.5 - 25 \cdots - 25]^T$.

We evaluate the convergence speed of potentials and data packets delivered to each sink node after traffic changes. To measure the convergence speed of potentials, we define the degree of the potential convergence $\epsilon_n(t)$ for each node that is given by

$$\epsilon_n(t) = \frac{|\bar{\theta}_n - \theta_n(t)|}{|\bar{\theta}_n|},$$



(a) Potential changes (no control)

(b) Potential changes (hierarchical control)

(c) Data packets delivered to sink nodes (no control)

(d) Data packets delivered to sink nodes (hierarchical control)

Figure 3. Potential and traffic convergence

where $\bar{\theta}_n$ corresponds to the target potential value of node n. We consider convergence to be achieved when $\epsilon_n(t)$ for all nodes becomes sufficiently small. Convergence time is defined as the minimum time taken by all sensor and sink nodes to satisfy the condition

$$\epsilon_n(t) < c, \tag{13}$$

where c is a constant set to 0.2.

Parameters $(\alpha, \beta, \sigma, r)$ are set to $(0.4, 0.2, 0.1, 10)$. Simulation result presented below is averaged over 30 simulation runs for each parameter setting.

B. Simulation Results

Figures 3(a) and 3(b) show the changes of potential values of the non-control scheme and our proposal, respectively. More exactly, these figures plot $\bar{\boldsymbol{\Theta}} - \boldsymbol{\Theta}(t)$ against time t. In these figures, thick lines correspond to potential changes of 4 sink nodes, and thin lines correspond to those of sensor nodes. Sink node potentials change more intensely than those of sensor nodes because sink nodes receive feedback inputs $\boldsymbol{u}$ directly from each sub-controller, whereas sensor nodes are indirectly affected by them via sink nodes.

As shown in Figs. 3(a) and 3(b), our proposal can enhance the convergence speed of potentials, as compared with the case of the non-control scheme. It takes 36,084 s for the potential convergence with the non-control scheme, while it takes 11,551 s with our proposal, although Figs. 3(a) and 3(b) show potential changes only within 10,000–30,000 s. As a result, potential convergence is accelerated by about 3.12 times due to our proposal. With the previous method [3], the convergence times is 10,453 s in average, which is approximately as short as our proposal, although the result is not shown in this paper. That indicates that the central controller precisely estimates degrees of interactions among sub-networks, and therefore, each sub-controller correctly estimates the potential dynamics of its corresponding sub-network and then returns control inputs for faster convergence of potentials although the sub-controller does not directly monitor any information of the other sub-networks. It is note that even if any controller fails, whole network can achieve convergence as long as nodes ignore feedback inputs such as Fig. 3(a) when self-organized control is properly designed. This means our proposal does not detract the robustness originated in self-organization.

From the point of view of the computational cost, the computational cost $O({N^i}^2)$ of sub-controllers for estimating the potential dynamics and calculating control inputs is much smaller than the computational cost $O(N^2)$ in [3] where only one controller monitors the potential dynamics of all nodes and then calculates control inputs for the entire network. Moreover, given the information of the network topology, the central controller is designed with the computation cost $O(N^2)$ as described in Subsection III-D, which is smaller than the computational cost $O(N^3)$ needed for designing the external controller proposed in [3]. Therefore, our proposal can enhance the convergence speed of potentials with the low computational cost even in large-scale networks, and, consequently, the hierarchical optimal feedback mechanism is more scalable than the non-hierarchical mechanism.

Figures 3(c) and 3(d) show the average number of data packets delivered to each sink node every 1,000 s. In each case, the number of data packets delivered to each sink node becomes disproportionate after the traffic changes at 10,000 s. Then sink nodes gradually become able to receive data packets equally because potentials are updated to adapt to the current flow rates. We can prove that the traffic convergence is also accelerated by hierarchical optimal feedback. This is because the potential convergence speed is enhanced by the hierarchical optimal feedback mechanism.

One problem we can find is that our proposal reduces the average number of data packets delivered to each sink node immediately after traffic changes. This problem originates from the optimal feedback mechanism (the same problem occurs in the previous mechanism [3]). In details, some sink nodes temporarily have the largest potential values within their communication ranges according to the control inputs, so data packets cannot arrive at sink nodes. Therefore, data packet would drop when the controller makes large changes to the potentials, while contributing to the faster convergence speed of potentials. However, the data packet drops are immediately reduced and the traffic finally converges faster than the non-control scheme because of the faster potential convergence. Note that in an actual situation data packets may be retransmitted instantly. Here, we evaluate only the case where data packets are never retransmitted because the main purpose of this paper is to reveal the upper limit of convergence speed of self-organizing systems.

In this evaluation, we show that our proposal enhanced the convergence speed of potentials with the much lower computational cost than the previous scheme [3]. That means that self-organization can be controlled for faster convergence while retaining the scalability which is an inherent and significant characteristic of self-organizing systems by introducing the hierarchical optimal feedback mechanism.

V. Conclusion and Future Work

We propose and evaluate potential-based routing with hierarchical optimal feedback, where two types of controllers, i.e., a central controller and sub-controllers, control the system in a hierarchical manner for faster convergence. Simulation result shows that the hierarchical feedback mechanism enhances the convergence speed of potentials without spoiling the scalability of self-organizing systems.

On the contrary, our proposal remains some challenging tasks. The communication delays for collecting the potential information are not considered in the evaluation of Section IV. Moreover, the optimal control improves the convergence speed of potentials but causes potential fluctuations as shown by simulation results. These fluctuations lead to data packet drops because sink nodes temporally have the highest potentials among their neighbors. There are tradeoff between the improvement of the potential convergence speed and potential fluctuations. Furthermore, the potential convergence is achieved as a result of the iterative behavior, i.e., the controller's optimal feedback and nodes' potential updates, so that potential cannot converge if environmental changes occur more frequently than the iterative behavior. We will investigate these points thoroughly in the future.

For future work, we will investigate controlled self-organization on interdependent networks, where several different networks interact with each other. Completely isolated networks rarely exist in the real world and networks are typically interconnected to the others like transportation networks, power grids and the Internet. Therefore, we need to consider influences of interactions among several networks for understanding and managing networks.

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References

[1] C.-M. Pintea, *Advances in bio-inspired computing for combinatorial optimization problems*. Berlin: Springer, Jun. 2014.

[2] M. Prokopenko, *Guided Self-organization: Inception*. Springer, Jan. 2014.

[3] N. Kuze, D. Kominami, K. Kashima, T. Hashimoto, and M. Murata, "Enhancing convergence with optimal feedback for controlled self-organizing networks," in *Proceedings of IEEE 80th Vehicular Technology Conference*, Sep. 2014, pp. 1–7.

[4] K. Zhou, J. C. Doyle, K. Glover *et al.*, *Robust and optimal control*. Prentice Hall New Jersey, Aug. 1995.

[5] D. Kominami, M. Sugano, M. Murata, and T. Hatauchi, "Controlled and self-organized routing for large-scale wireless sensor networks," *ACM Transactions on Sensor Networks*, vol. 10, no. 1, pp. 13:1–13:27, Nov. 2013.

[6] A. Sheikhattar and M. Kalantari, "Fast convergence scheme for potential-based routing in wireless sensor networks," in *Proceedings of Wireless Communications and Networking Conference (WCNC)*. IEEE, apr 2013, pp. 1980–1985.

[7] M. Koike, T. Ishizaki, T. Sadamoto, and J. Imura, "Hierarchical decentralized observer design and application to power networks (Japanese only)," *Systems, Control and Information*, vol. 27, no. 9, pp. 353–363, Sep. 2014.

[8] A. Antoulas, D. Sorensen, and S. Gugercin, "A survey of model reduction methods for large-scale systems," *Contemporary mathematics*, vol. 280, pp. 193–219, Oct. 2006.