Thermodynamics-based Strategy to Achieve Balance between Robustness and Performance for Self-organized Network Controls

Takuya Iwai, Masayuki Murata, Tetsuya Yomo
Graduate School of Information Science and Technology, Osaka University, Suita, Osaka 565–0871, Japan
Email: {t-iwai, murata, yomo}@ist.osaka-u.ac.jp

Daichi Kominami
Graduate School of Economics, Osaka University, Toyonaka, Osaka 560–0043, Japan
Email: d-kominami@econ.osaka-u.ac.jp

***Abstract*—Bio-inspired network controls are driven by the competition between their ordering force and disordering force. Both forces simultaneously affect their performance and robustness. Therefore, we must carefully determine their balance. In this paper, we focus on thermodynamic phenomena where a substance achieves the balance between both forces depending on its temperature. We translate bio-inspired network controls from the perspective of thermodynamics, and we analytically show that the appropriate balance between both forces can be achieved by selecting appropriate temperature.**

***Keywords*-bio-inspired network control; robustness; performance; balance; thermodynamics**

I. Introduction

Information networks must be more robust against ever-increasing dynamics and complexity. Many researchers have recently focused on interdisciplinary approaches to obtain innovative ideas. In particular, they have been actively working on self-organized network controls [1].

The self-organization [2] occurs under the competition between "ordering force" and "disordering force". The ordering force plays a role in organizing and maintaining useful structures for survival. The disordering force also plays a quite important role in diversifying the organized structures. In the field of self-organized network controls, the ordering force makes a network control change its state toward a better state. This force makes a significant contribution for achieving high performance. The disordering force makes its state veer toward an unintended state. This force plays an important role in achieving high robustness, which is a feature to prepare for unexpected failures, e.g. node failures. By achieving the appropriate balance between both forces, we can realize an excellent network control which achieves high performance and high robustness.

To achieve the appropriate balance between both forces, we focus on thermodynamic phenomena where a substance achieves the appropriate balance between both forces depending on its temperature. The objective in this paper is to show the validity of our approach to realize a network control which achieves the balance between its robustness and its performance. For this purpose, we firstly interpret network controls from the perspective of thermodynamics, and we formulate their thermodynamic state values, i.e. internal energy E_{nw}, entropy S_{nw}, temperature T_{nw}, and free energy A_{nw}. Next, we conduct a mathematical analysis and show that the appropriate balance between both forces is achieved by selecting the appropriate temperature T_{nw}.

II. Appropriate Balance between Ordering Force and Disordering Force

Taking a multi-path routing as an example of network controls, we formulate thermodynamic state values. Then, we analytically show that the network control can achieve the best balance between both forces by selecting appropriate temperature T_{nw} to its assumed network condition.

A. Mathematical Model of Multi-path Routing

There is a single pair of a source node and a destination node. For simplicity of mathematical analysis, we hereinafter assume that there is an infinite of disjoint path candidates. When the source node sends a packet to the destination node, the source node selects a path in a probabilistic manner. A path is identified by identifier x $(-\infty \leq x \leq \infty)$. The source node selects a path with smaller $|x|$ with higher probability. The probability for the source node to select path x is given by a Gaussian distribution function: $\exp(-x^2/2\sigma_2^2)/\sqrt{2\pi\sigma_2^2}$. When the source node selects path x, path x is disrupted with probability q_x $(0 \leq q_x \leq 1)$. This probability is hereinafter called "disruption probability" and is equally set at the same value q $(0 \leq q \leq 1)$ among all path candidates. Goodness of path x is represented by value G_x $(0 \leq G_x \leq 1)$. Goodness G_x is given by a Gaussian function: $1 - \exp(-x^2/2\sigma_1^2)$. As goodness G_x is smaller, path x is better, e.g. shorter hop length. When path x is disrupted, its goodness G_x is set at the worst value. In this case, its goodness G_x is set at 1.

B. Quantitative Definition of Thermodynamic State Values

1) Internal Energy E_{nw}: Internal energy E relates to the variability of its internal structure. In network controls, a state is maintained much more as it is better. In contrast, as a state is worse, it is more variable to search solution space for a better state. On the basis of this description, internal energy E_{nw} is quantified as "performance" of network controls. In case of the multi-path routing, internal energy E_{nw} is measured as the expected goodness of a path, which the source node selects in a probabilistic manner. Therefore, internal energy E_{nw} is formulated by the next equation.

$$E_{nw} = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\sigma_2^2}} e^{-\frac{x^2}{2\sigma_2^2}} \left\{ (1-q) \left(1 - e^{-\frac{x^2}{2\sigma_1^2}} \right) + q \right\} dx$$

Here, internal energy E_{nw} becomes smaller, that is, its performance becomes higher as disruption probability q becomes smaller, variance σ_1^2 becomes larger (that is, there are a more number of good paths), or variance σ_2^2 becomes smaller (that is, the source node selects a path from a narrower range of path candidates).

2) Entropy S_{nw}: Entropy S relates to the randomness of its internal structure. A network control sometimes changes its state toward unintended states, and it can tolerate occurrences of unexpected failures, e.g. node failures. On the basis of this description, entropy S_{nw} is quantified as its "robustness". In case of the multi-path routing, the probability for the source node to select path x is given by the Gaussian distribution function. Therefore, entropy S_{nw} is formulated as the entropy of the Gaussian distribution with variance σ_2^2 and average 0.

$$S_{nw} = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\sigma_2^2}} e^{-\frac{x^2}{2\sigma_2^2}} \log \frac{1}{\sqrt{2\pi\sigma_2^2}} e^{-\frac{x^2}{2\sigma_2^2}} dx$$

Here, entropy S_{nw} becomes larger, that is, its robustness becomes higher as variance σ_2^2 becomes larger, that is, the source node selects a path from more diverse path candidates.

3) Temperature T_{nw}: In thermodynamics, temperature T is defined by dE/dS. In case of the multi-path routing, we differentiate internal energy E_{nw} and entropy S_{nw} with respect to variance σ_2^2, and we can derive temperature T_{nw} from equation $T_{nw} = (dE_{nw}/d\sigma_2^2)/(dS_{nw}/d\sigma_2^2)$. Therefore, temperature T_{nw} is formulated as the next equation.

$$T_{nw} = (1-q) \left\{ \frac{1}{\sigma_2\sqrt{\sigma_1^{-2}+\sigma_2^{-2}}} - \left(\frac{1}{\sigma_2\sqrt{\sigma_1^{-2}+\sigma_2^{-2}}} \right)^3 \right\}$$

4) Free Energy A_{nw}: In thermodynamics, free energy A is defined by equation $A = E - T \times S$. In network controls, free energy A_{nw} is defined by the next equation.

$$A_{nw} = E_{nw} - T_{nw} \times S_{nw}$$

Through self-organization, the network control reaches steady state, where the balance between internal energy E_{nw} (performance) and entropy S_{nw} (robustness) is well kept depending on temperature T_{nw}. Therefore, free energy A_{nw} captures the imbalance between its performance and its robustness.

C. Numerical Example of Analytical Model

Through a mathematical analysis, we show the appropriate balance between both forces is achieved by selecting the appropriate temperature T_{nw} which depends on the assumed network condition.

Figures 1 and 2 represent numerical examples about temperature T_{nw} and free energy A_{nw}, respectively. Disruption probability q of all paths is set at 0.2. We prepare two scenarios. In the first scenario, variance σ_1^2 of the Gaussian function, which determines the goodness of paths, is set at 10 assuming that there are a small number of good paths. In the other scenario, variance σ_1^2 is set at 30 assuming that there are a large number of good paths. In these figures, x-axis corresponds to variance σ_2^2 of the Gaussian distribution which determines probability p_x for the source node to select path x, and y-axis corresponds to temperature T_{nw} and free energy A_{nw}, respectively.

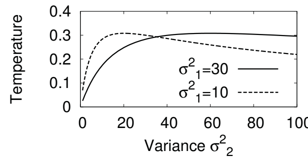

Figure 1. Temperature

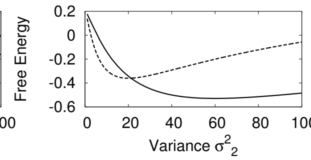

Figure 2. Free energy

We can find that variance σ_2^2 which maximizes temperature T_{nw} is equal to one when free energy A_{nw} is minimized by comparing Fig. 1 with Fig. 2. Given the network condition, i.e. a certain disruption probability q and a certain variance σ_1^2, we can expect that the maximization of temperature T_{nw} leads to achieving the appropriate balance between the ordering force and the disordering force. To prove this, we first derive variance σ_2^2 when temperature T_{nw} is maximized. For this purpose, we differentiate temperature T_{nw} with respect to variance σ_2^2. Now, we get equation $\sigma_2 = \sigma_1/\sqrt{2}$. Next, we differentiate free energy A_{nw} with respect to variance σ_2^2. We substitute condition $\sigma_2 = \sigma_1/\sqrt{2}$ into the result, and we get zero. Consequently, we conclude that variance σ_2^2 when free energy A_{nw} is minimized is the same with variance σ_2^2 when temperature T_{nw} is maximized. This implies that we can derive the appropriate parameter setting from the formulation of temperature T_{nw}.

III. Conclusion and Future Work

In this paper, we translated the multi-path routing from the perspective of thermodynamics. Then, we analytically showed that the appropriate balance between both forces can be achieved by selecting the appropriate temperature. As future work, we are planning to verify our approach using a more realistic model of network controls and propose a thermodynamics-based design method.

Acknowledgement

This research was supported by Grant-in-Aid for Fellows of Japan Society for the Promotion of Science (JSPS).

References

[1] C. Zheng and D. Sicker, "A survey on biologically inspired algorithms for computer networking," *IEEE Communications Surveys Tutorials*, vol. PP, pp. 1–32, Jan. 2013.

[2] E. Bonabeau, M. Dorigo, and G. Theraulaz, *Swarm intelligence: from natural to artificial systems*. Oxford University Press, 1999.