An entropy-based energy consumption analysis of a network

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***Abstract*— The energy consumption by the Internet is rising at an unprecedented rate. Future networks should be able to provide the means for meeting this energy demand. Numerous researches have been carried out to understand the energy consumption level of a network. However, there are few approaches to develop a metric that indicates and analyzes the energy consumption level of a network based on its topological structure.**

In this paper we propose an entropy-based metric that can be applied for the evolution of energy efficiency in the future Internet. With the proposed entropy-based metric, we analyze the energy consumption levels of various networks that have different topological structures. Moreover, we demonstrate that the entropy-based metric has a positive linear relation with the energy consumption level of a network with intensive simulation studies.

***Index Terms*— Energy consumption, Entropy, Future networks.**

I. Introduction

Energy issues of future networks have come to the fore in recent years. Many studies estimate that power consumption of the Information and Communication Technology (ICT) sector rises up to 10% of the world wide power consumption according to [1] [2]. In addition, around 8% of the total energy in the U.S. was consumed by network components of the Internet, and it would grow by 50% within a decade [3]. Thus, in near future, the Internet could be constrained on its continued growth due to this energy restriction.

Thus, there have been several research attempts to investigate this energy conservation issues. As a pioneering work, the authors in [4] proposed the idea of sleeping of network components for energy conservation in the Internet. To achieve the goal, they suggested to construct a topology that has a good correlation between the number of awake devices and traffic load. Thus, a network can be operated partially depending on traffic load. In [5], some experiments were carried out to investigate where most power is consumed in a network. They found that the amount of energy consumption by a network component is nearly constant and independent from the actual traffic load. Thus, they suggested to maximize the utilization of network components to minimize power consumption level of a network. In [6][7], the authors investigated how many nodes and links can be turned off while maintaining the global connectivity of the network as well as sustaining Quality of Service (QoS)[1]. They proposed a heuristic algorithm to find the minimum set of nodes and links satisfying the given traffic demand and QoS, and reported that 23% of total energy conservation can be easily achieved. Another optimization approach was proposed to find the minimum energy consumption level of an IP over WDM networks in [8]. They reported that minimizing the number of IP router ports can potentially and maximally save the energy consumption of a IP over WDM network. Their observation is matched to the result presented in [5] which minimizing the number of used network components is the best way to reduce energy consumption of a network.

[1]E.g., maximum link utilization under 50%

All of the related works summarized above focused on the energy consumption level of a network using either experiments or optimization approaches. Especially, searching the minimum set of network components (routers and line cards) that consume the least energy is quite straightforward since the problem is equivalent to the designing of the cost minimized network. However, comparing to these approaches, developing a metric or an indicator that measures the level of energy consumption level of a network has not been investigated in other literatures to the best of our knowledge.

In this paper we attempt to define an entropy-based metric that indicates the energy consumption level of a network. Entropy was originally introduced to describe potential energy level of a system in both the classical thermodynamics [9] and statistical mechanics [10]. Thus, considering an entropy-based metric to understand the energy consumption level of a telecommunication system was an intuitive approach. Some backgrounds of entropy are given in the next Section. We firstly derive the positive linearity relation between the energy consumption level and entropy of a network analytically. Then, the derived relation is verified numerically. With the developed entropy-based metric, we consider how topological structure of a network affects its energy consumption level. For this purpose, we construct networks that have different topological structures, and show how entropies of these networks vary due to their topological structures. To estimate the real energy consumption level of a network, we formulate a Linear Programming (LP) formulation of the problem that is to search the minimum total energy consumption level of a network. Moreover, we investigate how the energy consumption level of a network is affected by the energy consumption behavior of network devices.

The rest of this paper is organized as follows. In Section II, we derive the relation for a telecommunication between its energy consumption level and its entropy. This is followed by numerical evaluations of the derived the relation in Section III. Finally, we conclude the paper in Section IV.

II. Derivation of the relation for a network between its energy consumption level and entropy

The term entropy was coined by Rudolf Clausius [9] in the context of classical thermodynamics. According to the definition, entropy of a system can be interpreted as potential energy that the system contains. The classical thermodynamics was extended to the new field called statistical mechanics by James Clerk Maxwell and Lugwig Boltzman [10]. In classical thermodynamics, energy is associated with a single whole system. One the other hand, in statistical mechanics energy is described by the role of the tiny particles of the system.

In statistical mechanics, entropy represents the state of whole system (e.g., temperature called macrostate) with the states of all tiny particles (e.g., directions of particles called microstate) in the system. For instance, the energy level of a system (e.g., gas) becomes maximized when all molecules of a gas have very different states (e.g., direction). Due to this reason, the entropy is also interpreted as a measure of disorder since entropy increases as the states of microstates become close to random.

The mathematical foundation of statistical mechanics were applied into new academic field called the modern information theory by claude shannon [11]. He adopted entropy to measure the level of uncertainty associated with probabilistic events.

There have been a lot of arguments to define the concept of entropy clearly since it can be interpreted in many different ways as shown above. Since entropy was originally introduced to describe potential energy level of a system in both the classical thermodynamics and statistical mechanics, it is a natural choice to consider an entropy-based metric to measure the energy consumption level of a telecommunication system.

In addition, we found an attempt in the field of hydrology community that entropy was used as an indicator of the potential energy level of a river network in [12]. With the retranslation of the potential energy of a river network in the context of a telecommunication, we adopted an entropy-based metric to define the relation between the energy consumption level of a telecommunication network and its entropy.

The potential energy of a river network can be considered as the amount of water that the river network contains. In terms of telecommunication networks, it can be translated into the amount of traffic that a network accommodates. Since a telecommunication network consumes more energy to accommodate more traffic load, their approach can be extended to telecommunication networks for defining the relation between the energy consumption level of a telecommunication network and its entropy.

The derivation begins with the understanding of the principle of maximum entropy estimate approach[13]. Maximum entropy estimate provides a quantitative technique for assigning probabilities using currently available information, and strictly avoiding the use of any other information that we may use unconsciously. As the name implies, it is an estimation method, in other words, when given constraints are not enough (under-determined problem) to determine the exact solution of an optimization problem, it selects the most plausible solution by maximizing entropy of the system.

Keeping in mind the principle of maximum entropy estimate approach, the optimization problem shown in Equ. (1)(2)(3) can be formulated to make an estimation of a probability distribution p_i.

$$\text{Maximize} \quad S = -\sum_{i=1}^{D} p_i \log p_i \tag{1}$$

subject to

$$\sum_{i=1}^{D} p_i e_i = E \tag{2}$$

$$\sum_{i=1}^{D} p_i = 1 \tag{3}$$

To understand the notations of the optimization problem, let us have a look Fig. 1. Assuming that traffic travels down from node A (This node is called the outlet node.) into the end nodes. p_i is the probability that a randomly chosen link is at i^{th} hops from the node A. e_i represents the energy which is consumed by a flow when it reaches to i^{th} hops from the node A. It is natural to assume that a network consumes more energy when traffic flows travel more hops in the network. For this reason, E represents the average energy consumption level of the network. D is the longest distance from the outlet node to end nodes. In this example, D is equal to 4.

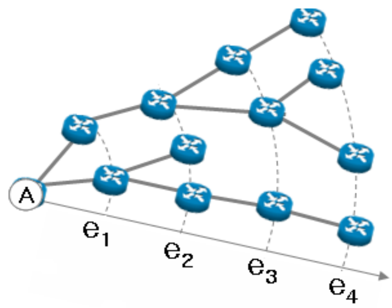

Fig. 1. A schematic example.

The solution of the optimization problem is known as the Maxwell-Boltzman distribution, and is shown as follows,

$$p_i = e^{-\mu e_i} / \sum_{i=1}^{D} e^{-\mu e_i} \tag{4}$$

where μ is the lagrangian multiplier. After taking log of both sides of Equ. (4), it becomes

$$e_i = -\frac{1}{\mu}\log p_i \sum_{i=1}^{D} e^{-\mu e_i}$$
$$= -\alpha \log p_i \beta, \quad (\alpha = 1/\mu, \quad \beta = \sum_{i=1}^{D} e^{-\mu e_i}) \quad (5)$$

Now, by substituting e_i in Equ. (2) with Equ. (5),

$$E = \sum_{i=1}^{D} p_i(-\alpha \log p_i \beta)$$
$$= -\alpha \sum_{i=1}^{D} p_i \log p_i - \alpha \log \sum_{i=1}^{D} p_i \beta$$
$$= \alpha S - \alpha \log \beta \quad (6)$$

In [12], the authors defined α as DT where T represents the temperature of the network. Since temperature does not mean much in the telecommunication network, we ignore it so that we assume that α represents only D.

Finally, Equ. (6) shows the relation for a telecommunication network between its energy consumption level (E) and entropy (S). Considering that α and β are constant values, it shows the positive linear relation between its energy consumption level (E) and entropy (S). In other words, the higher entropy value a network has, the more energy it consumes.

III. Evaluations

A. Verification of the relation between its energy consumption level and entropy shown in Equ. (6).

Initially, we setup the following simulation scenario. Barabási and Albert (BA) [14] and Erdos and Renyi (ER) [15] topologies (N=1000, m (average degree)= 3, 4, ..., 30) are constructed. Then, their minimum cost spanning trees (MST) are found after the costs of individual links are set using the Equ. (7). It is because the entropy we propose here was introduced for a river network which has a tree structure.

$$c_{ij} = 1/k_i k_j \quad (7)$$

where k_i is the degree of node i. Due to the Equ. (7), links belonging to high degree nodes tend to have low costs so that they are likely to be selected as part of minimum spanning tree (MST). For this reason, the MSTs of BA and ER topologies obtained using the equation can maintain the topological properties of the original BA and ER topologies such as degree distribution. In addition, to select the outlet node (node A in Fig. 1) of a network, we select a node with the highest betweeness centrality in the network. Finally, the entropies (S) and the energy consumption levels (E) of the BA and ER MSTs are calculated using Equ. (1) and Equ. (2), respectively. Their relation is plotted in Fig. 2.

We observe the clear positive linear relations between the entropies (S) and the energy consumption levels (E) in both figures. The data sets are fitted by the linear equation shown in Equ. (6), and the estimates of coefficients α and β of the equation are shown in Table. 1. The α value of ER is larger than that of BA. As mentioned previously, α represents D which shows the longest distance from the outlet node to end nodes. Since ER topology tends to have longer D than BA[2], the former is likely to have larger α than the latter.

In addition, β is in inverse proportion of the sum of e_i[3], thus, if we assume that a traffic flow consumes same energy per hop, a network with large β consumes more energy than a network with small β. Thus, it suggests that BA topology consumes less energy than ER topology. We can also observe the superiority of BA topology to ER in terms of energy saving by comparing the entropy values of both topologies.

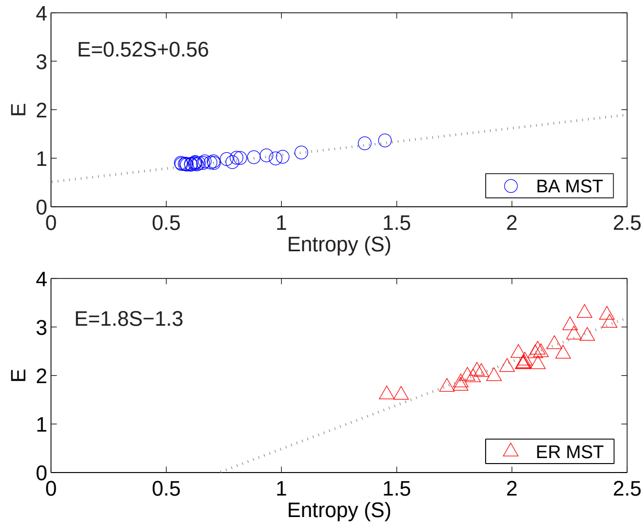

Fig. 2. The energy consumption level (E) versus the entropy (S) of BA (Top) and ER (Bottom) minimum spanning tree (MST) topologies.

Table 1. Estimates of coefficients α and β in Fig. 2.

	BA MST	ER MST
α	0.52	1.80
β	0.47	1.65

B. Verification of the linearity relation in different types of topologies

Here, we investigate the relation between the energy consumption levels and the entropies in different types of networks other than BA and ER topologies. For this purpose, we initially construct ER topologies and transform their topological structures to be power law topologies[4] using the rewiring method in [16]. In the middle of the topological

[2]Because ER has generally longer average shortest path (ASP) than BA.

[3]The energy which is consumed by a flow when it reaches to i^{th} hops from the outlet node.

[4]Degrees (links) of all nodes in a network are characterized as a distribution function $P(k)$ that is the probability that a randomly chosen node has degree k. When the degree distribution of a network follows a power function P(k)$\sim k^{-\gamma}$, it is called a power law topology.

evolution, we observe different types of topologies whose degree distributions are shown in the top figures on the left side of Fig. 3.

This result also shows the clear positive linear relation between the energy consumption levels of the topologies and their entropies. Additionally, it demonstrates that the topological transformation proposed in the paper[16] can be utilized to evolve a topology into an energy efficient topology.

C. Investigation of the energy consumption level of a network.

In the previous sections, we demonstrated that BA topology (power law topology) is more energy efficient than ER topology since the entropy value of the former is smaller than that of the latter. In this section, we verify this observation by investigating the energy consumption levels of BA and ER topologies with realistic simulation scenarios.

1) Simulation I: Initially, BA and ER topologies (N=200, m≈2(directional topology)) are constructed. For statistical purpose, 15 sets for each case are generated. All links of the topologies have 60 units of capacities and consume 1 unit of energy per 1 unit of flow[5]. Each topology has total 39800 origin-destination (OD) pairs[6]. Among the whole OD pairs, only (R = 1%, 4%, 7%, ... 22% increased by 3%.) of them are randomly activated. A demand for each OD pair is generated from the uniform distribution [0, 1], and 10 different paths are set between each OD pairs in terms of its energy consumption level. Then, the energy consumption level of a network is formulated as a multi-commodity flow problem. Firstly, let us define the notation in Table. 2.

Table 2. Notations.

Notations	Explanation
K	Traffic demands. Each demand is shown as k (1≤ k ≤ K).
$\mathbb{P}^k$	A collection of directed paths from the origin node to the destination node.
$f(P)$	The flow on a path P.
$e(P)$	The energy consumption of the path P.
$\delta_{ij}(P)$	Equals one if a link (i, j) is contained in the path P and is zero otherwise.
e_{ij}	Per unit power consumption of flow on the link (i, j), e.g., $e(P) = \sum_{(i,j)\in P} e_{ij}$. The link (i, j) connects two ports i and j. We assume that the two ports consume same amount of power.
c_{ij}	The capacity of each link(port). The link (i, j) connects two ports i and j. We assume that the two ports have same capacities.
d^k	The size of the k^{th} demand for all k= 1, . . . , K.

With the above notations, we modify the path-flow formulation in Ahuja et al [17] to provide an Linear Programming (LP) formulation of the problem that is to search the minimum total energy consumption level of a network. It is shown as follows,

$$\text{Minimize} \quad \sum_{1\le k\le K} \sum_{P\in\mathbb{P}^k} e(P)f(P) \tag{8}$$

subject to

$$\sum_{1\le k\le K} \sum_{P\in\mathbb{P}^k} \delta_{ij}(P)f(P) \le c_{ij} \tag{9}$$

$$\sum_{P\in\mathbb{P}^k} f(P) = d^k \tag{10}$$

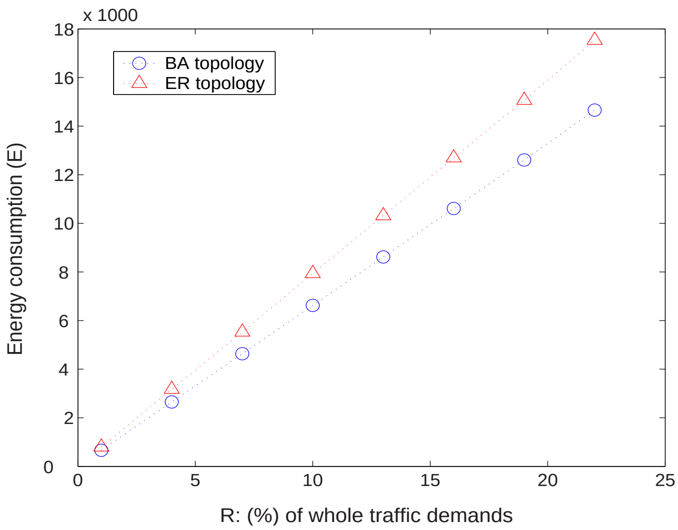

Fig. 4. The optimized energy consumption levels of both BA and ER topologies (N=200, m≈2) are calculated by increasing the numbers of activated OD pairs in both topologies. The 95%confidence intervals are small enough to be inside the marks so they are omitted. Each result is averaged over 15 realizations.

This LP formulation searches a set of routes for given traffic demands that consume the least energy. The constraints in Equ. (9) restrict the amount of flow on a particular link (port) (i, j). The last constraint in Equ. (10) shows the sum of all flows between one OD(Origin-Destination) pair k must be equal to the given demand d^k. To solve this LP problem, GLPK [18] (GNU Linear Programming Kit) was used, which is a free software package intended for solving large-scale linear programming (LP), mixed integer programming (MIP), and other related problems.

In Fig. 4, BA topology consumes less energy than ER topology since the latter increases more rapidly than the former. This result is well matched to the one that we predicted in Fig. 2.

2) Simulation II: In this simulation, we observe how the energy consumption levels of BA and ER topologies vary when the energy consumption model [19] is applied for both topologies.

The authors [19] introduced a metric called the energy proportionality index (EPI) that quantifies the proportionality between the consumed energy and the traffic load on a network device. This proportionality behavior of network device

[5]The link can accommodate 60 units of flow at maximum, and the energy consumed by the link is same as the amount of flow on the link.

[6]The number of OD pairs in a network with N nodes is equal to N*(N-1).

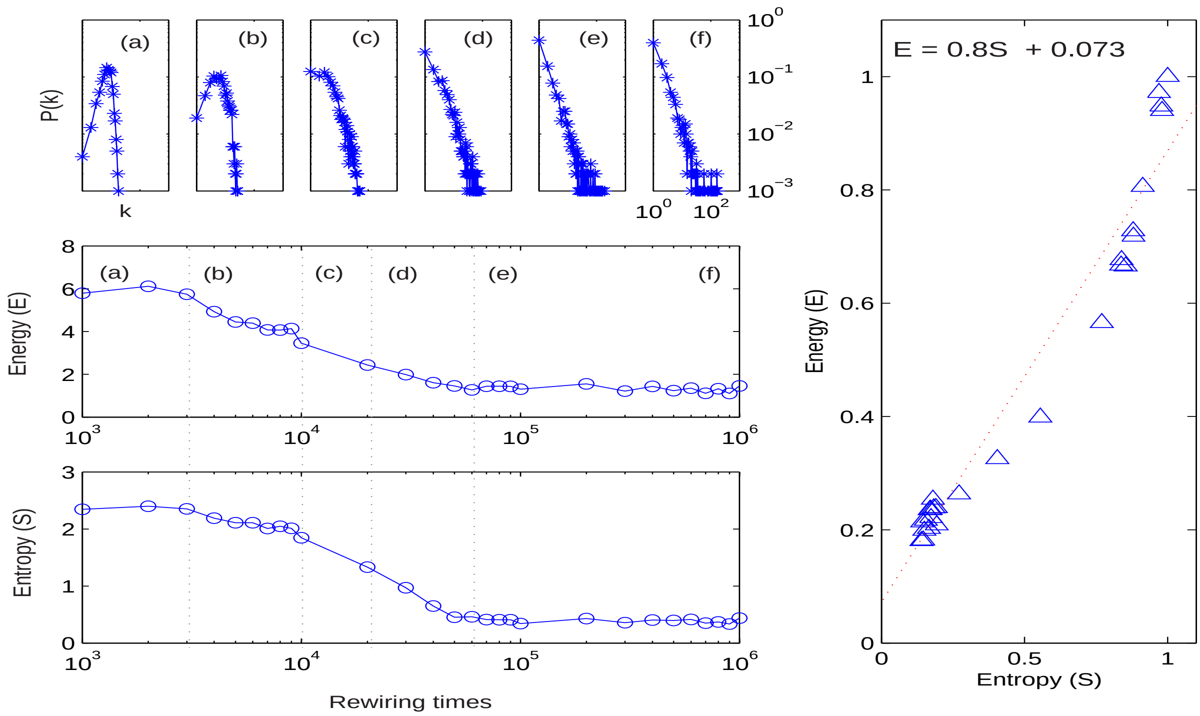

Fig. 3. Top figures on the left side show the degree distributions of ER topology (N=1000, m=4) at different times of being rewired (e.g., (a): $T = 10^3$, . . . (f): $T = 10^6$). The rewiring process was conducted on ER topologies based on our previous paper[16]. The below two figures show the energy consumption levels (E) of the minimum spanning trees (MST) of the counterpart topologies and their entropies (S). Lastly, the right figure plots the relation between the energy consumption levels (E) and the entropies (S) after individual elements are normalized by its respective maximum element. Each result is averaged over 10 realizations.

is required to make future networks more energy efficient [20] [21]. The energy proportionality index (EPI) is defined as follows,

$$EPI(\%) \quad = \quad (1 - I) * 100 \tag{11}$$

where I represents the energy consumption level of a device when it is under idle (no traffic) condition. With 100% EPI, the device consumes energy perfectly proportional to traffic load on the device. On the other hand, a device with 0% EPI consumes constant energy regardless of the amount of traffic passing through the device.

With the same simulation setup described previously, we calculate the total energy consumption levels of BA and ER topologies using Equ. (12).

$$TCE \quad = \quad \sum_{i}^{E}[(1 - I) * u_i + I] \tag{12}$$

where TCE stands for total consumed energy, E is the total number of links, and u_i represents the utilization of link i.

Fig. 5 plots the energy saving rate of BA comparing to ER topology as EPIs of individual devices in both topologies increase. When EPI is equal to zero, the energy consumption levels of both BA and ER topologies are same since both topologies have same number of links. Remember that a link consumes constant amount of energy regardless of traffic load

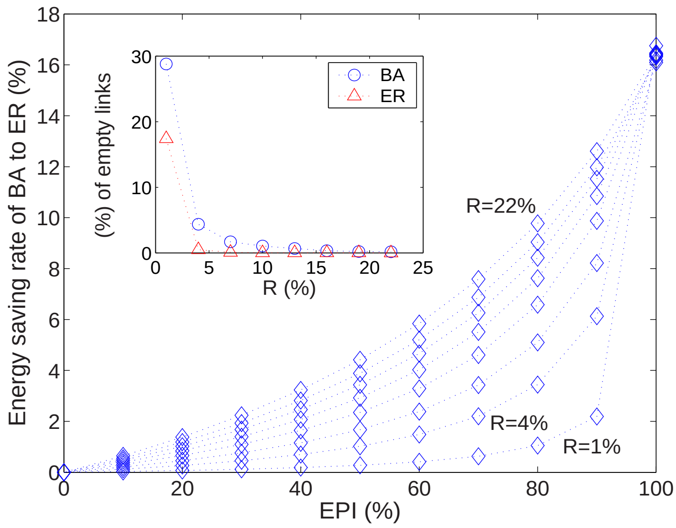

Fig. 5. Energy saving rate of BA comparing to ER topologies as EPI varies. R represents the percentage of the activated OD pairs. The inset figure shows the percentage of empty links (links with no traffic load) as the number of activated OD pairs increases. The inset figure shows the number of empty links (link with no traffic load) as the number of activated OD pairs increases in both BA and ER topologies. Each result is averaged over 15 realizations.

when EPI is equal to zero (I=1). As the energy saving rate of BA to ER increases as EPI increases. It implies that

BA topology can benefit more from the network devices that consume energy proportional to traffic load than ER topology does. In addition, the differences among the energy saving rates at different R values suddenly shrink at EPI=100% since the number of empty links (link with no traffic load), which is related with the value of R (Refer to the inset figure), does not affect on the energy consumption level any more.

IV. Conclusions

In the classical thermodynamics and statistical mechanics, entropy has been used to describe potential energy level of a system. By adopting the idea of entropy, we developed an entropy-based metric that measures the energy consumption level of a telecommunication network. We demonstrated that the relation between energy consumption level and entropy of a network shows a positive linear relation, and the linearity relation was verified with intensive simulation studies.

Initially, we showed that BA is a more energy efficient topology than ER by analyzing the entropy values of both topologies. To confirm the observation, we investigated the energy consumption levels of BA and ER topologies with realistic simulation setups. Moreover, we also demonstrated that the proposed entropy metric can predict the energy consumption levels of various types of networks also.

For the understanding of energy consumption behavior of a network, the proposed entropy-based metric mainly focused on the topological structure of a network without considering several important factors such as traffic demand matrix. We leave the inclusion of traffic demands as a parameter into the entropy-based metric for our future works.

Acknowledgments

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