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On Attractor Perturbation through System-inherent Fluctuations and its Response

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Abstract—The recent trends in communication technology toward an ambient information network society will inevitably lead to an increased diversity and heterogeneity of devices, as well as network traffic. Efficient cooperation and interaction among connected devices and protocols must be able to cope with a large degree of fluctuations. In this paper we discuss the application of the attractor perturbation concept from biological systems for utilizing inherent traffic fluctuations to smoothly control the access rate at a gateway node injecting traffic into the core network. This adaptive control scheme is performed only by using instantaneous observations of the local buffer occupancy and its variance.

1. Introduction

One of the main purposes of *ambient networking* [1] is to provide a seamless method for cooperation and interaction among different types of access networks (e.g., wireless mesh networks or cognitive radio) connecting to a backbone core network and offering the user custom-made services. However, due to the large heterogeneity among possibly connected devices and protocols, it is expected that a large traffic volume subject to a high level of fluctuations will be traversing through these gateway nodes. Since gateway nodes separate the local traffic from global traffic, a careful and moderate control must be performed regarding their access. While not too much traffic should be injected into the core network, the control should not be made in a strict way, which would lead to timeouts and packet drops, but rather by subtle and small influence utilizing the inherent fluctuations in the local traffic pattern.

In order to accomplish this, the concept of *attractor perturbation* can be utilized, which provides a mathematical relationship between the inherent fluctuations of a system and its response. The principle of attractor perturbation states that a “force”, which is input to a fluctuating system, will result in a linear relationship in the system's response depending on the degree of its fluctuations, given by its variance. Our study proposes the control of the traffic rate at a gateway node's input buffer in an ambient network infrastructure without explicitly knowing the traffic characteristics. This is done by only using measurements of the time series of a fluctuating observable quantity, in this case the buffer occupancy (see Fig. 1). Through the concept of attractor perturbation, the rate of the local traffic can be controlled by applying a subtle force in such a way that the end-users in the local network do not become aware of it and that it is most beneficial for the whole system.

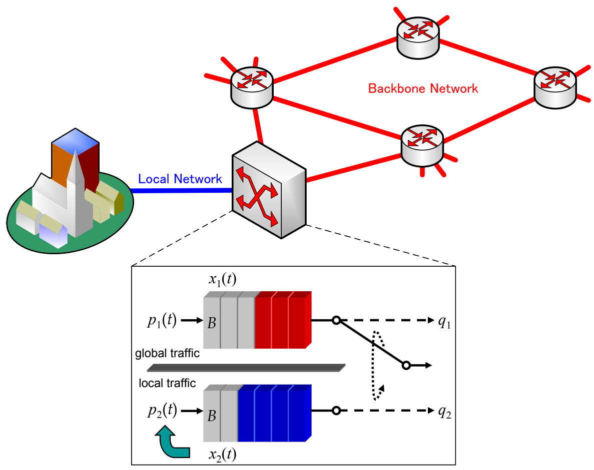


Figure 1: Buffer at access node in an ambient network

2. The Attractor Perturbation Concept

It is evident that in biological organisms and cells there are always some inevitable fluctuations in quantity or behavior [2]. In fact, the existence and exploitation of fluctuations is one of the key features of *self-organizating systems* [3]. Our approach is based on the work of Sato et al. [4], where a general relationship between fluctuation and response in biological systems is provided. It was also discussed later in [5].

We can summarize the general concept as follows. Consider a measurable *variable* x (e.g. concentration of a protein), which is subject to fluctuations, and which exists within a biological *system* (e.g. cell or organism). Furthermore, let us denote a *parameter* a, which influences the system and can be externally controlled (e.g. DNA sequence of a gene). Since the variable $x(t)$ is a time-dependent quantity, we can observe its time series and obtain its distribution without explicitly knowing the influence of the parameter a on x. Especially, the average μ_a

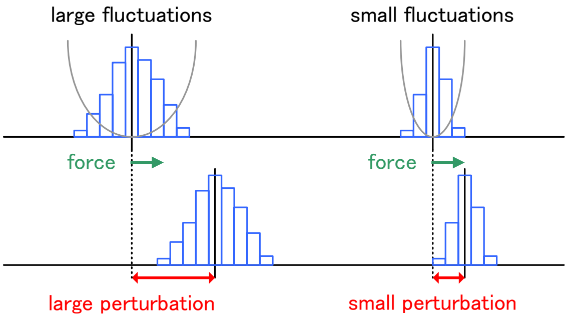


Figure 2: Basic principle of attractor perturbation

and variance σ_a^2 under the condition of parameter a can be obtained. It is shown in [4] that for a change in parameter from $a \rightarrow a + \Delta a$ the resulting perturbation in the average value of x is linearly proportional to the variance prior to the force σ_a^2 with b being a constant as shown in Eqn. (1).

$$\mu_{a+\Delta a} - \mu_a = b\,\Delta a\,\sigma_a^2 \tag{1}$$

The relationship in Eqn. (1) is theoretically derived for a unimodal Gaussian-like distribution and verified experimentally by observing the distribution of fluorescence proteins after genetic evolutions of a bacteria in [4]. The basic principle of this concept is illustrated in Fig. 2. By applying the same force through the control parameter a, the variable x that has a larger variance can be perturbed more easily than when its variance is small. Note that this relationship does in no way assume any explicit knowledge on how a and x are related and, thus, provides a good control method in systems with unknown dynamics.

2.1. The Relationship between Attractors and Probability Distributions

Equation (1) actually only describes the stochastic properties of the variable x. In this section, we discuss more clearly how this is related to the concept of attractors and dynamic systems in order to show why this feature can be considered as stable mechanism.

Let us consider the simple case of a one-dimensional dynamic system given in Eqn. (2).

$$\frac{d}{dt}\,x(t) = -\rho\,(x(t) - x_0) + \eta(t) \tag{2}$$

The term $\eta(t)$ represents the background noise, whereas ρ defines the softness of internal control within the system. Here, we have an attractor at x_0. The total noise in the system is, thus, given by two factors $\eta(t)$ together with ρ, which are in general unknown quantities. Defining this total variance as $\sigma^2 = D/\rho$, where D is half of the variance of η and assuming that the resulting probability distribution can be expressed by a normal distribution, we have the probability density function as in Eqn. (3),

$$p(x) = p(x_0)\exp\left(-\frac{(x - x_0)^2}{2\sigma^2}\right) \tag{3}$$

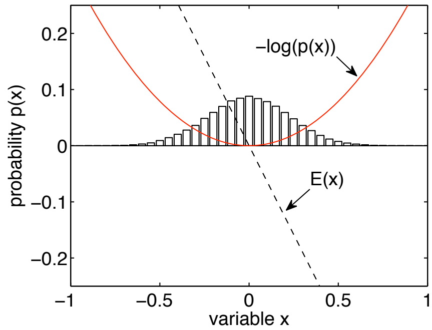


Figure 3: Fluctuating system with loose control ($\rho = 0.1$)

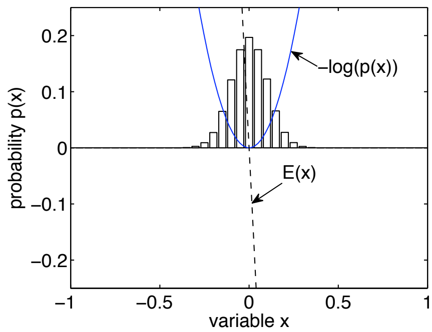


Figure 4: Fluctuating system with tight control ($\rho = 1.0$)

where

$$p(x_0) = \frac{1}{\sqrt{2\,\pi\,\sigma^2}}. \tag{4}$$

By taking the negative of the logarithm on both sides of Eqn. (3), we obtain the following equation.

$$F(x) = -\log(p(x)) = \frac{(x - x_0)^2}{2\,\sigma^2} - \log(p(x_0)) \tag{5}$$

Thus, we can characterize an energy potential function by the derivative over x as

$$E(x) = \frac{dF}{dx} = -\frac{\rho\,(x - x_0)}{D}. \tag{6}$$

Figures 3 and 4 illustrate the above derivation for $2D = 0.1$, $x_0 = 0$. In both figures, the background noise value remains constant, but in Fig. 3 the system has a more loose control by having a smaller ρ value. Figure 3 shows the histogram, its logarithmic transformation, as well as the resulting potential function as a dashed line. On the other hand, Fig. 4 allows less fluctuations, which is indicated by a steeper energy potential and can be considered as a more stable and tightly controlled system.

2.2. Dynamics of One-Dimensional System

Let us investigate the attractor perturbation dynamics by applying a force Δa to perturb the state value x. In our context a force is a constant term, which shifts the equilibrium

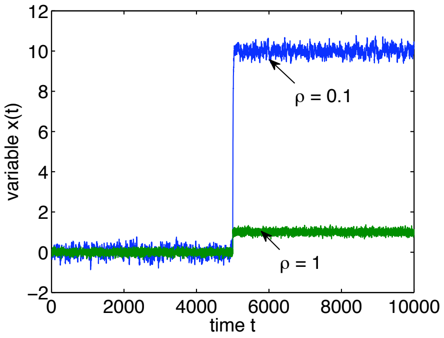


Figure 5: Time series of fluctuating variable $x(t)$ after applying the same force

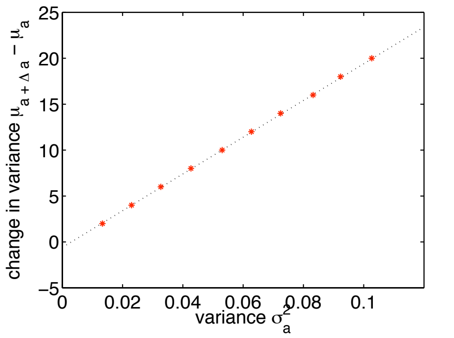


Figure 6: Illustration of the linearity in response to fluctuations

point away from it's initial value x_0. In the experiments shown in Fig. 5, the numerical simulations run from $t = 0$ to $t_{max} = 10000$. At time $t = t_{max}/2$, a "unit force" of $\Delta a = 1$ is added, shifting the mean value of the state x. We can see that adding the same force results in a shift of the average for different degrees of internal fluctuation ρ. The tightly controlled system with $\rho = 1.0$ shows the least influence, whereas for $\rho = 0.1$ we have a significant increase in mean from $\mu_a = 0$ to $\mu_{a+\Delta a} = 10$. In general, applying a constant force to the system results in a larger perturbation for higher fluctuating systems and the offset is linear in proportion to the variance of the observed variable before applying the force. This linear relationship is illustrated in Fig. 6.

3. Proposed Rate Control Mechanism

We now propose a rate control mechanism based on the attractor perturbation concept for the gateway node scenario in Fig. 1.

3.1. Description of the Algorithm

Let us assume that the gateway node has two buffers, one for global traffic and another for local traffic. Each buffer has a maximum capacity of B. Furthermore, let us assume that the system operates in discrete time steps, so that packet interarrivals are geometrically distributed and depend on the packet arrival probabilities p_1 (global traffic) and p_2 (local traffic). At the output port, there is a work-conserving scheduler that multiplexes both traffic streams. We assume that the arrivals at each queue are independent and we can only observe the time series of the buffer occupancy at the local buffer $x_2(t)$. Our objective is to utilize the attractor perturbation concept to control the local traffic arrival probability $p_2(t)$, which can be considered as the control parameter for the attractor perturbation method.

The basic algorithm for controlling the local arrival rate can be outlined as follows.

1. Measure the local buffer occupancy values over a sliding window W to obtain the time series $x_2(t - W), \ldots, x_2(t)$.

2. Calculate the variance over the measured values as

$$v_2(t) = \mathrm{Var}[x_2(t - W), \ldots, x_2(t)] \quad (7)$$

   A method for efficiently maintaing the variance for a sliding window of a data stream is for example given in [6].

3. Update the local arrival probability $p_2(t)$ as follows

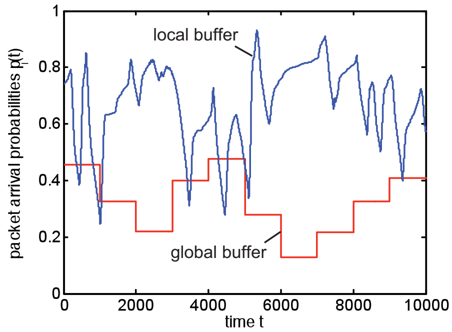
$$p_2(t + 1) = p_2(t) + \delta(t)\, s(t) \quad (8)$$

   where $\delta(t) = \mathrm{sign}(B/2 - x_2(t))$ is the direction in which the force is applied and $s(t) = f(v_2(t))$ is the strength of adaptation utilizing the linear relationship given by Eqn. (1).

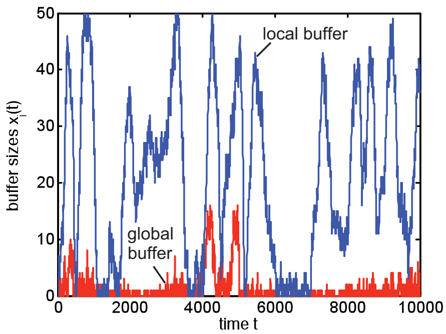
3.2. Results from Numerical Experiments

The numerical results from a simulation experiment are shown in Fig. 7. In this experiment, we randomly choose a new arrival probability p_1 for the global buffer every 1000 time steps as shown in Fig. 7(a). Based on the scheduling discipline and the arrival probabilities, the local buffer size will fluctuate over time as can be seen in Fig. 7(b). Note that we also show in this figure the occupancy of the global buffer $x_1(t)$, but we assume that this information is hidden from the view of our proposed buffer control scheme. Based on the time series of $x_2(t)$, it is possible to extract the variance $v_2(t)$ of the local buffer occupancy over a sliding window as shown in Fig. 7(c).

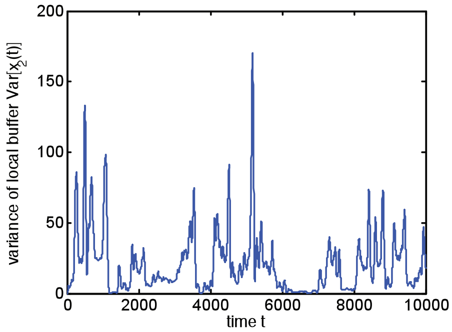
We now apply the attractor perturbation concept by setting the adaptation strength as $s(t) = v_2(t)/B^2$ in order to control the local buffer arrival probability $p_2(t)$ in Fig. 7(a). This figure also illustrates that $p_2(t)$ reacts roughly inversely to the arrival probability of the global buffer. Furthermore, the local buffer level $x_2(t)$ fluctuates much more, which indicates that the total flow of packets is nearly in balance. If the inflow is less than the outflow, both buffer levels would be low, but the system would be underutilized. On the other hand, if the inflow is larger than the outflow,



(a) Adaptation of local traffic arrival rate



(b) Evolution of buffer sizes over time



(c) Variance is used for computing the adaptation strength

Figure 7: Example of time series for rate control

at least one of the buffers would be near the buffer capacity, leading to increased blocking and packet loss.

4. Conclusion

In this paper we discussed about the relationship between system-inherent fluctuations and the response, which we coined under the term as the attractor perturbation concept. By taking this method into account, a fluctuating system can be influenced in a controlled and subtle way. Beside the theoretical formulation and comparison between attractor systems and probability distributions, we also provided an application to the rate control at the buffer of a gateway node in an ambient network environment. Simple numerical experiments showed that although this method is rather simple, it is capable of adapting to changes in the environment. In the future, we are planning to extend this method to a higher dimension, as well as study the applicability of our proposal to further network applications and services. Our goal is to apply the benefits of attractor perturbation to the dynamics of the attractor selection scheme [7, 8].

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