Methods to Exploit Noise in the Design of Complex Systems

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Abstract: Since biological organisms have evolved in a noisy environment, it is likely that they have intricate mechanisms in place to deal with noise, and even to exploit it. Engineers can learn much from biology, and to this end we briefly review some methods in which noise can be used in systems in an advantageous way. Among these methods are: *stochastic search*, *encoding and transmission of information by noise*, *stochastic resonance*, *fluctuation-enhanced sensing*, and *synchronization through noise*. This paper gives some examples in which these mechanisms are expected to improve the efficiency of engineered systems.

Keywords: noise, biological organisms, design of systems

1. Introduction

Noise is a common phenomenon in the world around us, and scientists and engineers have learned to live with it through a variety of techniques, the most common of which is suppression to increase the Signal-to-Noise ratio (SNR). Other techniques have also been developed, like the use of Error Control Codes to encode information in a redundant way, such as to ensure that its original contents can be recovered in case of data corruption. These traditional techniques, however, have their limitations, which are particularly notable in areas like integrated electronics, because of the need to decrease signal levels with increased levels of integration. When scales are in the range of nanometers, noise takes on a major role, and it is quite remarkable that biological organisms can cope so well with it. Molecular motors, for example, which play an important role not only in cellular transport, but also in muscular movements, typically work in an environment full of Brownian motion and still manage to conduct their tasks at a remarkable efficiency. Brain activity is another example in which noise appears to have only limited negative effects on the tasks being conducted. What are the secrets of these natural feats and can engineers use similar mechanisms when designing complex systems?

In this paper, we review methods that aim to exploit noise rather than to be impeded by it. We take inspiration from biological organisms, but also provide some examples of systems driven by noise for which no biological counterpart is available.

2. Stochastic Search

Stochastic search refers to a process that conducts a random walk in a space of possible states with the aim of settling in one (usually optimal) state.

2.1. Molecular Motors

Though the phrase “stochastic search” is mostly used in the framework of algorithms in computer science, it also plays an important role in processes taking place on molecular scales. In this context the above-mentioned random walk originates in the Brownian motion of particles, which is caused by the collisions of molecules to these particles. *Molecular motors* are an example of biological mechanisms that exploit the properties of Brownian motion [Astumian94, Yanagida08]. Typically, a molecular motor has an arm that conducts a random walk over a number of potential binding sites, to eventually settle for a site satisfying certain conditions (see Fig. 1).

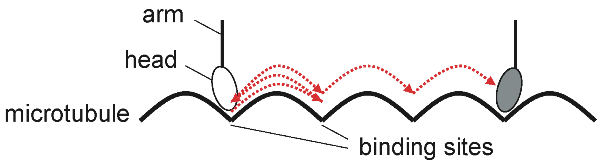

Fig. 1: Principle of a molecular motor. A molecular arm is driven by noise over a number of binding sites, until it finds one that matches certain conditions.

Such conditions may for example be the slope of the potential energy of the arm being in a certain range. The binding of the arm to a site will be kept intact until the arm has moved in a certain direction, after which the arm is released from the binding site. Called *Brownian Ratchet*, this mechanism biases motion in a certain direction and is widely thought to offer a good explanation of how chemical energy is efficiently transferred to mechanical energy in biology.

2.2. Token-based Circuits

Brownian search has recently been proposed [Peper09, Lee09] in the context of token-based circuits, which are circuits in which signals have a discrete character, like Petri-nets [Girault03]. Traditional token-based circuits tend to suffer from deadlocks, because circuit elements requiring multiple tokens as their input will fail to conduct operations as long as not all of their inputs are available. To avoid the resulting deadlocks, special functionality is required in the circuit elements, which increases their complexity. Brownian search offers an alternative, since it can be used to backtrack out of deadlocks, thereby making unnecessary deadlock-avoiding functionality in the circuit elements. The resulting circuits are remarkable simple as compared to traditional token-based circuits [Peper09]. A recently constructed cellular automaton based on this principle can achieve computational universality, even though its cells are remarkably simple [Lee08]. Token-based circuits have some potential to be realized as nanometer-scale technologies, in which signals take the form of individual particles, like in *Single Electron Tunneling* circuits [Likharev87, Nazarov92].

2.3. Attractor Selection

Stochastic search has also been explored for routing algorithms in networking [Leibnitz06]. Inspired by the dynamics of gene regulatory networks, this scheme is an extension of the scheme in [Kashiwagi06] such as to be able to dynamically adapt to changes in the system environment through dynamically switching between deterministic behavior and random search. This switching behavior is controlled by the activity in the system. Due to the formulation as a system of stochastic differential equations, no explicit rules are needed in this scheme, and convergence towards attractors is achieved, even if a noise term is present. The principle of attractor selection is outlined in Fig. 2.

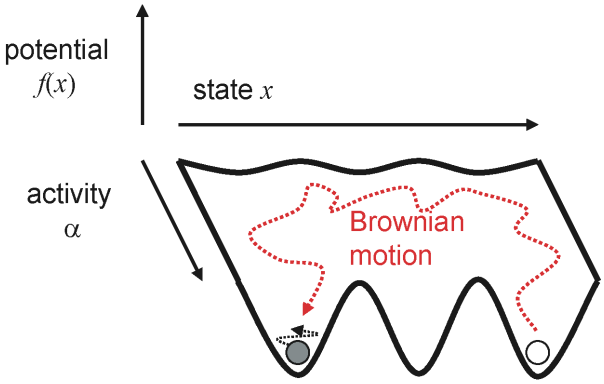

Fig. 2: Graph illustrating attracter selection. The horizontal axis represents the system internal state x, which is for simplicity only shown in the one-dimensional case. In this case there exist three attractors and the orbit of the system is currently at the rightmost attractor. If this solution is no longer suitable due to the influence of some environmental factor, the activity function decreases, leading to a “flatter” potential landscape defined by f(x) on the vertical axis. Given that the noise term is unchanged, the system will be driven away from its current attractor and perform a random walk in the phase space. Once the system approaches a more suitable attractor (in this case the one at the left), the system will become more deterministic, staying at this attractor in spite of the still existing noise terms.

Although the *attractor selection scheme* is not necessary optimal in terms of performance, it is very robust to fluctuations and changes in the environment [Leibnitz09].

3. Information Encoding and Transmission

3.1. Noise in the brain

The brain has a remarkable ability to process information, and it is no surprise that many researchers have investigated the mechanisms underlying this ability. Key in this context is the scheme used to encode information in the brain. It is well-known that neural cells transmit information to each other through the firing of sequences of spikes, but there is still controversy on how this information is exactly encoded. It has long been known that the rate at which spikes are fired tends to strongly correlate with the intensity of an input stimulus. There is, however, significant variability in the spike amplitudes and the so-called *interspike intervals* (ISI). A central question is whether this variability is part of the signal or whether it is just noise [Stein05]. It may be due to the variability at which signals arrive in some sensory systems, such as the irregular intervals (shot noise) at which photons arrive at retina cells. The auditory system has more precise timing of spikes due to the temporal character of its inputs, but even in this system there is noise caused by Brownian motion in sensory hair cells [Harris68, Corey83]. It has also been pointed out that some of the variability in spike timing may originate in the stochastic nature of synaptic transmission [Stein05]. What are the advantages of the variability in ISI? Noise could enhance the ability of a system to measure a signal’s wave form, rather than just its frequency [Stein05]. It could also automatically tune the system’s sensitivity when the signal is weak according to a phenomenon called *Stochastic Resonance* (see Section 4).

3.2. Utilization of Noise in Communication

The use of noise in encoding information has also been considered in engineered systems. An example is the *Code Division Multiple Access* (CDMA) technology [Viterbi95] widely found in today’s 3rd generation wireless systems (3G). In CDMA, each user’s data signal is spread over a large frequency spectrum by modulating it with a unique spreading code. These codes are designed to be orthogonal to each other in order to provide maximum separation among each connection. On the uplink, all users use pseudo-random codes for simultaneously transmitting their data to the base station, so each signal is perceived as noise on the wireless channel making it hard to detect or eavesdrop on transmissions. On the downlink, the base station uses fully orthogonal codes for channelization. Using noise-like carriers is especially beneficial in wireless communication, since the signal strength is constantly subject to fluctuations due to fading. Since each user contributes to the total interference in the cell, it is important to maintain a minimum SNR, which leads to a “soft” capacity definition for CDMA-based systems [Veeravalli99].

Another use of noise to encode information has recently been proposed in [Kish08]. In this scheme, which is loosely inspired by the stochasticity of brain signals, different levels of signals (like a zero or a one) are encoded through different independent noise sources. The orthogonality of these sources allows the design of binary or multi-valued logic circuitry based on analog circuitry. Potential advantages are reduced power consumption, increased robustness to noise and variability problems, though the scheme also has challenges, such as an overhead in time and hardware resources due to the necessity to decode signals before operations.

Finally, a remarkable use of noise to encode information, found in [Kish05, Kish06, Kish07], is *secure encryption* based on classical physics. Rather than using quantum physics, as in competing schemes, this proposal is based on Kirchhoff's law applied to the circuit that includes the communication wire. The idea is that two different physical states of the system can only be distinguished between by the two actors at both sides of the wire – the sender and the receiver – and not by an outside observer, unless the latter conducts an active measurement on the system, which can be detected. Both the sender and the receiver have at their side a switch that can select between a low resistance, denoted by 0, and a high resistance, denoted by 1 (see Fig. 2).

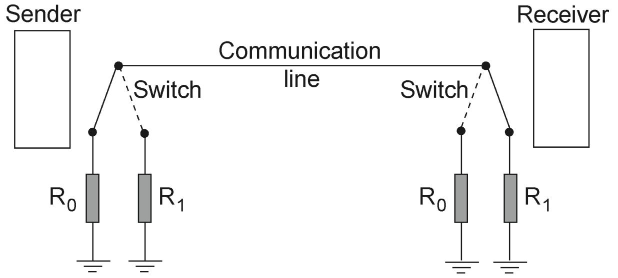

Fig. 3: Kirchhoff-loop-Johnson(-like)-noise communicator achieves secure communication by using Kirchhoff's law and Johnson-noise.

There are four states of the system, i.e., 00, 11, 01, and 10, and the latter two states give the same combined resistance of the circuit, thus being undistinguishable to the outside observer, and only these two states will be used to encode information. Noise is used in this framework to measure the combined resistance of the circuit by the sender and the receiver without giving more information than that to an outside observer, through the property that resistances generate so-called *Johnson-Nyquist* noise that is proportional to the resistance. The practical realization of this scheme requires noise generators to generate sufficient levels of noise in proportion to the switch settings, since Johnson-Nyquist noise itself has insufficient levels for long communication wires to be practical. Compared to quantum encryption, this classical noise-based method is superior in terms of security, robustness, integrability on a chip, and cost-effectiveness.

4. Stochastic Resonance

Stochastic resonance (SR) [Benzi81] is the addition of noise to a sub-threshold signal to improve the signal transfer and SNR through a nonlinear system. It is often used in bistable systems, in which the level of a signal itself is insufficient to drive a transition from one state to the other (see Fig. 4). Typically, the added noise to the system is of a level just sufficient to make the resulting signal exceed a threshold, but not much larger, as it would obscure the signal. Without the additional noise, one would obtain few transitions, with a random character. It has been noted that adding a high-frequency AC signal instead of noise has a similar or better effect on the signal propagation, though this may cause unavoidable inferences due to frequency beats [Kish07].

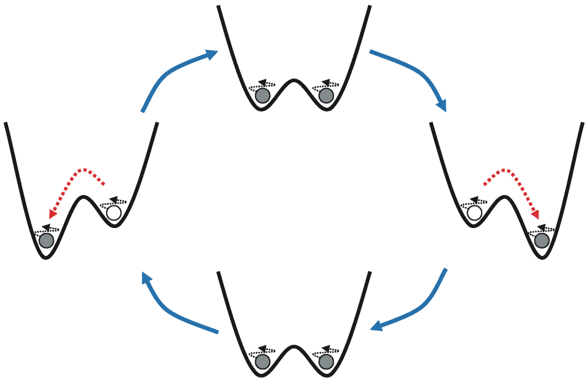

Fig. 4: Principle of Stochastic Resonance. In a bi-stable system, noise will facilitate the transition of a system into a lower energy state.

SR has been shown to play an important role in the amplification of weak biological signals [Hanggi02, Moss04] and propagation of neural signals [Chapeau-Blondeau99, Balazsi00]. It was discovered in sensory neurons that are subject to external noise [Moss04]. Since such neurons operate as threshold systems, they are very suited in this framework, due to their intrinsic noisiness. Especially interesting is the automatic gain control by SR via the firing dynamics of neurons, which results in the relaxation threshold potential being reset to a high level after firing [Gingl00].

It is important to realize that SR can enhance SNR and signal gain *through the given nonlinear system* [Godivier98, Kish01], but it *cannot* provide greater information channel capacity than its actual value at the input of the nonlinear system. It only enhances the performance of the nonlinear system, which in itself can be useful [Rousseau05].

Conventional SR assumes signals that are sub-threshold most of the time, but it has been discovered that SR can also be applied with signals that are much larger, i.e., supra-threshold. Early signs of this were observed in systems using supra-threshold spiky signals, like neural spikes, which resulted in a significantly enhanced SNR [Kish96, Loerincz96]. Supra-threshold SR has also been analyzed in systems consisting of an array of identical nonlinear devices, each of which receives an independent but identically distributed noisy signal and one common deterministic signal [Stocks00]. After the signals pass through the devices, they are summed, to produce the output of the system. It turns out that this arrangement gives an increased SNR if the number of devices is large enough. The improvement results from the diversity induced by the injected noises, in the responses of the individual devices of the array [Rousseau04]. Applications of supra-threshold SR include signal estimation tasks [Rousseau03], motion detectors [Harmer01], and cochlear implants [Stocks02].

The main purpose of supra-threshold SR is to transfer a signal from input to output with a minimum of distortion. In other words, the system should behave linearly. It has been observed in the past that noise, if of sufficiently high level, has the tendency to linearize certain types of systems. The reason for this phenomenon may be rooted in the fact that a signal passing through a nonlinear dynamic system will be affected in different ways at different frequencies. Adding noise to the signal will spread this deformation over a larger range of frequencies, and if the noise level is sufficiently high, the system will start to behave like a linear system, leaving the basic form of the signal intact [Dykman94]. SR in the usual linear response limit (large noise small signal), however, cannot increase SNR [Dykman95]. The opposite of linearization, i.e., delinearization, may also take place based on similar principles, when systems can be tuned to one or more of their eigenfrequencies through the injection of noise.

5. Fluctuation-Enhanced Sensing

Fluctuation-enhanced sensing (FES) [Bruschi94, Kish00] is a method of sensing in which the stochastic component of a sensor signal is used to extract information additional to that from the sensor alone. Rather than using the average signal from a sensor, the use of the power density spectrum, for example, allows a strongly enhanced sensitivity, due to the fact that the power spectrum is a pattern, not a single value. Other statistics that have been proposed are higher-order statistics of skewness and kurtosis [Schmulko04]. Since noise is frequently the most sensitive parameter in a sensor, FES is often capable of sensing a significant increased variety of signals, with enhanced sensitivity and selectivity, and it has been successfully applied in chemical sensing [Schmera08]. FES may even produce a response to chemical agents in cases where a classical sensor signal does not [Hoel02]. The use of noise to enhance the ability of a system to measure a signal’s wave form, as is mentioned in Section 3.1, is also a form of fluctuation-enhanced sensing.

6. Synchronization through Noise

When multiple nonlinear dynamic systems receive input from a common noise source, the noise may act as to weakly couple the systems, and make them synchronize. In the framework of the brain, this may underlie the synchronicity at which neurons fire synchronously. Sensory neurons lying near each other, for example, receive common stimuli, of which noise is one component. The resulting synchronous behavior may be meaningful in biological information processing such that neurons generate a collective response [Zhou03]. Synchronicity may also be an important factor to ensure that learning through synaptic strengthening takes place, since correlated firing of presynaptic and postsynaptic neurons is a necessary condition for learning, according to the well-known principle laid out by Hebb that neurons that fire together wire together [Hebb49].

Though common noise – and by extension synchronization through common noise – is a construct that is unlikely to occur in many practical situations, it is claimed to be closely related to generalized synchronization through mutual interaction [Guan06]. Synchronization through common noise is unlikely to be achieved by the constant bias of noise, but rather by the noise driving the systems into convergence regions, where the differences between nearby trajectories shrink quickly [Lai98]. Possible applications of synchronization through common noise include secure communication [Minai98]. The idea is to subject two chaotic oscillators at respectively the sender and the receiver to common noise over a public channel, which will then synchronize the oscillators, allowing them to generate identical output, which could be used as the basis of an encryption key. Recovery of this key by an eavesdropper would be very complicated due to the chaotic nature of the oscillators. Note that this method to use noise for encryption differs from the noise-based encryption based on the Kirchhoff-loop and Johnson noise in Section 3.2.

7. Conclusions

Noise plays an important role in nature, and it is no surprise that researchers and engineers have attempted to exploit it in order to achieve designs with improved properties. This paper has given a brief overview of the use of noise in biological systems and engineered systems. It is expected that the trend towards increased use of noise in designs will continue, driven by the further miniaturization of electronics and microelectromechanical systems.

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