

Stability Analysis of Window-Based Flow Control Mechanism in TCP/IP Networks

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Abstract

Recently, a new version of the TCP mechanism, called TCP Vegas, is proposed, and has potential to achieve better performance than current mechanisms such as TCP Tahoe and Reno. This paper considers a window-based flow control mechanism based on a slightly modified congestion avoidance mechanism of TCP Vegas, and introduces its mathematical model described by a nonlinear difference equation. We calculate its fixed point, and a stability condition of the fixed point is derived.

1 Introduction

Explosive growth of computer networks has brought severe congestion problems[1], and reactive flow control and congestion control is needed to regulate transmission of information from source hosts to destination hosts. Many mechanisms such as window-based control[2] and rate-based control[3] have been proposed for avoiding congestion, and a feedback configuration is essential to provide efficient data transfer services[4, 5, 6, 7, 8]. It is expected that control theory gives a new insight in congestion control, and its applications have been studied. Keshav[4] proposed a stable control scheme using fuzzy logic based estimators. Fendrick[5] formulated delayed feedback scheme, and analyzed its stability. In ATM networks, many control-theoretic approaches have been found in [9, 10, 11, 12, 13]. Moreover, a game theoretic approach[14, 15] and a hybrid approach combining a traditional control technique and fuzzy neural networks[16] are also discussed. However, almost all control-theoretic approaches have adopted rate-based congestion control mechanisms.

On the other hand, a window-based flow control mechanism called TCP(Transmission Control Protocol)[17] has been widely used in current packet-switched networks such as TCP/IP networks. There are several versions of TCP such as Reno, Tahoe, and Vegas, and performance evaluations of these versions have been

reported[18, 19, 20, 21, 22]. TCP Reno implemented in BSD UNIX uses packet losses in the networks as feedback information, and controls a window size[17]. Recently, TCP Vegas has been proposed in order to improve performance such as throughput[18, 19, 20], and 37-71% improvement of total throughput and about 1/5-1/2 reduction of the number of retransmitted packets have been achieved. A major reason for such good performance is an introduction of a new congestion avoidance mechanism.

In this paper, we discuss stability of a window-based flow control based on TCP Vegas. Section 2 introduces a mathematical model of a network with a window-based flow control mechanism. Section 3 gives a stability condition of a fixed point in the proposed model.

2 Mathematical Model

We consider a network model shown in Fig. 1 where N source hosts are connected to the corresponding destination hosts through one router. We introduce a mathematical model of the network. Let $w_n(k)$ be a current window size of the host n ($1 \leq n \leq N$) at slot k , and $q(k)$ the number of packets stored in router's buffer. Moreover, let L and B be the buffer's capacity and the bandwidth of the router, respectively. We assume that each source host n has always send $w_n(k)$ packets at

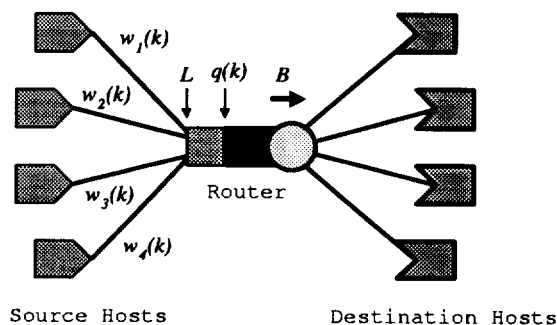


Figure 1: Network model

each slot, and that the round trip times of all connections are equal. Then, the dynamics of the buffer is given by the following nonlinear difference equation:

$$q(k+1) = \min \left(\max \left(\sum_{n=1}^N w_n(k) - Br(k), 0 \right), L \right) \quad (1)$$

where $r(k)$ is the round trip time at slot k .

We review a congestion avoidance mechanism of TCP Vegas briefly. Refer to [19] for more detail. We assume that each source host knows a round trip time (RTT) τ in the case that there is no congestion in the network. In practice, τ is set to be the minimum of all measured RTT's. Note that τ corresponds to the round-trip transmission delay when the buffer is empty, that is, τ is equal to the sum of all propagation delays and processing delays at the router. Then, the throughput is expected to $w_n(k)/\tau$ if the network is not congested. On the other hand, in practice, we can obtain the RTT $r(k)$ at slot k by measuring time delay from sending a packet to receiving its ACK packet, and it is given by the following equation in our mathematical model.

$$r(k) = \tau + \frac{q(k)}{B} \quad (2)$$

The current throughput is $w_n(k)/r(k)$, and the error $d_n(k)$ between the expected and the current throughput is calculated as follows:

$$d_n(k) = \frac{w_n(k)}{\tau} - \frac{w_n(k)}{r(k)} \quad (3)$$

Vegas controls the next window size $w_n(k+1)$ based on the error $d_n(k)$ and given control parameters α and β as follows:

$$w_n(k+1) = \begin{cases} w_n(k) + 1 & \text{if } d_n(k) < \alpha \\ w_n(k) - 1 & \text{else if } d_n(k) > \beta \\ w_n(k) & \text{otherwise} \end{cases} \quad (4)$$

Intuitively, these parameters α and β correspond to having too little and too much extra data in the network, respectively.

From Eq. (4), Vegas does not change the window size if $d_n(k)$ lies in the interval $[\alpha, \beta]$, which sometimes causes unfairness among the connections[22]. So, in our analysis, in order to improve the fairness, we introduce a new control parameter γ , and set $\gamma = \alpha = \beta$, and Eq. (4) is modified as follows:

$$w_n(k) = \max(w_n(k) + \delta(\gamma - d_n(k)), 0) \quad (5)$$

where δ is a control parameter in order to determine the change rate of the window size per each RTT. So it is regarded as a feedback gain.

3 Stability Analysis

For simplicity, we assume that all the initial window sizes of all hosts are equal, and all hosts change their window sizes according to Eq. (5). Then the queue length $q(k+1)$ of the buffer in the router at slot $k+1$ is given by

$$q(k+1) = \min(\max(Nw(k) - Br(k), 0), L) \quad (6)$$

where

$$w(k) \equiv w_n(k) \quad 1 \leq n \leq N$$

Moreover, by Eq. (3), the error $d(k) \equiv d_n(k)$ ($1 \leq n \leq N$) is given by

$$d(k) = \frac{w(k)}{\tau} - \frac{w(k)}{r(k)} \quad (7)$$

We use the following approximation.

$$Br(k) \simeq B\tau$$

Then, using Eqs. (5), (6), and (7), a fixed point (w^*, q^*, d^*) of the considered network is given as follows:

$$\left. \begin{aligned} w^* &= \tau \left(\frac{B+\gamma N}{N} \right) \\ q^* &= \gamma N \tau \\ d^* &= \gamma \end{aligned} \right\} \quad (8)$$

The linearized equation around the fixed point is given by

$$x(k+1) = Ax(k) \quad (9)$$

where

$$x(k) = \begin{bmatrix} w(k) - w^* \\ q(k) - q^* \end{bmatrix},$$

$$A = \begin{bmatrix} 1 - \frac{\delta}{\tau} + \frac{B\delta}{(B+\gamma N)\tau} & -\frac{B\delta}{N(B+\gamma N)\tau} \\ \frac{B\delta}{(B+\gamma N)\tau} & 0 \end{bmatrix}$$

Using a stability test such as Jury's test[23], it is easily shown that the fixed point is locally exponentially stable if and only if the following inequalities hold.

$$\delta > 0 \quad (10)$$

$$\frac{\delta(B-\gamma N)}{(B+\gamma N)\tau} + 2 > 0 \quad (11)$$

$$\frac{B\delta}{(B+\gamma N)\tau} < 1 \quad (12)$$

Thus, we have the following conclusion.

- If the number N of connections is greater than $3B/\gamma$, then the upper bound of the control parameter δ is given by Eq. (11). So, as N increases, the upper bound decreases monotonically, and converges to 2τ . Therefore, in this case, the network is always stable if δ is set to be 2τ .

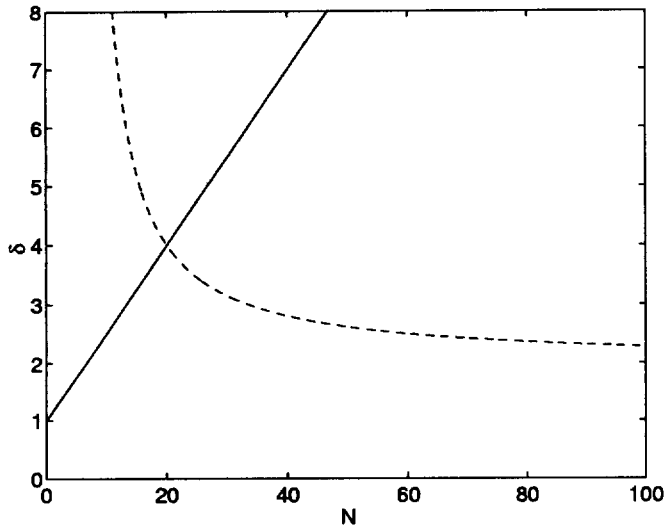


Figure 2: Stability region in the δ - N plane

- If N is less than $3B/\gamma$, then the upper bound is given by Eq. (12). So the upper bound increases linearly as N increases. Since δ is bounded by τ at $N = 0$, the network is always stable if δ is set to be τ .

Shown in Fig. 2 is a stability region where $B = 20$ [packets/ms], $\tau = 1$ [ms], and $\gamma = 3$. Note that $B = 20$ corresponds to 163.8[Mbit/s] if the packet length is equal to 1 [Kbyte]. In this figure, the dotted and solid lines correspond to Eqs. (11) and (12), respectively. When the number N of connections are less than 20, δ can be taken larger as N increases. But when it is greater than 20, δ should be smaller as N increases. It is shown that the network is always stable for any $N > 0$ if δ is less than one. Moreover, since the dotted line converges to $\delta = 2$, for sufficiently large number of connections, we take δ less than 2 for the network to be stable. Equation (5) is regarded as static feedback with feedback gain δ from the control theoretical point of view. So, in general, if δ is set to be larger, that is, we use high gain feedback, then transient behaviors of the network will be better while stability will be worse. Figure 2 shows such a tendency.

Shown in Figs. 3 and 4 are behaviors of the window size and the queue length for $N = 10$ when the network are stable and unstable, respectively. When the network is stable, both the window size and the queue length converge to the fixed point given by Eq. (8). However, when it is unstable, they oscillate with large amplitude, which implies that transmission delay of packets becomes very large. Moreover, the buffer becomes empty periodically, which implies that the throughput of the network degrades. Thus, it is concluded that δ has to be selected such that the network is stable.

This paper derived a stability condition of a window-based flow control mechanism based on a slightly modified congestion avoidance mechanism of TCP Vegas. In the case that the number of connections is small, a control parameter can be set to be larger as the number of connections increases while the reverse feature occurs in the case that the number is large.

It is a future work to extend our analysis to more realistic networks. For example, both TCP and UDP traffic coexist in real TCP/IP networks, and interference between different types of traffic should be taken into consideration.

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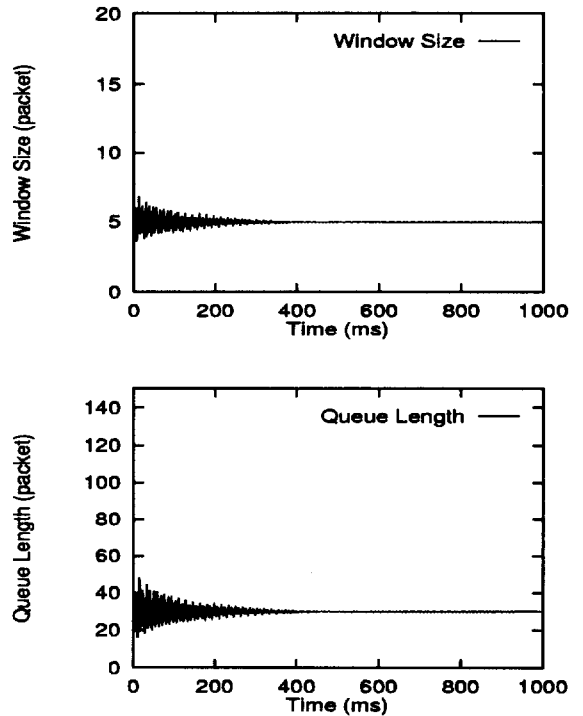


Figure 3: Stable behavior($\delta = 2.4$)

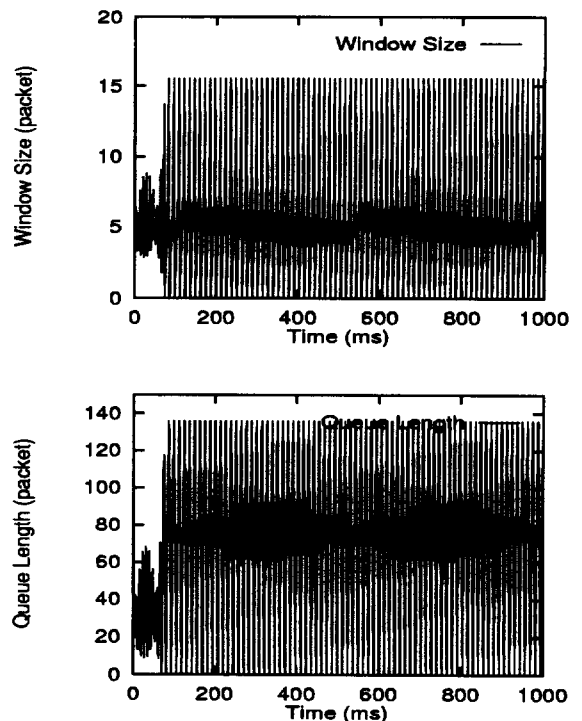


Figure 4: Unstable behavior($\delta = 2.6$)